

## PRACTICE SET 4: RATES, RATIOS, PROPORTIONS, AND PERCENTAGES

### *Rates, Measurement, and Unit Conversions*

By now, you've become adept at using algebra to answer many SAT math questions, which is great because you'll need those algebraic skills to answer questions involving rates and other quantitative reasoning questions. You're likely already familiar with all kinds of rates—kilometers per hour, meters per second, and even miles per gallon are all considered rates.

Most rate questions can be solved using some form of the *DIRT equation*—Distance = rate  $\times$  time. If you have two of the three components of the equation, you can easily find the third.

Units of measurement are also important when answering rate questions (and others that require a unit conversion). The *factor-label method* is a simple yet powerful way to keep calculations organized and to ensure that you arrive at an answer that has the requested units.

For example, suppose you need to find the number of cups in 2 gallons. To use the factor-label method, start by identifying the initial unit (gallons), and then identify the desired unit (cups). The next step is to piece together a path of relationships (measurement conversions) that will convert gallons into cups, cancelling units as you go. Keep in mind that you will often have multiple stepping-stones along the way:

$$2 \cancel{\text{ gallons}} \times \frac{4 \cancel{\text{ quarts}}}{1 \text{ gallon}} \times \frac{2 \cancel{\text{ pints}}}{1 \cancel{\text{ quart}}} \times \frac{2 \text{ cups}}{1 \cancel{\text{ pint}}} = (2 \times 4 \times 2 \times 2) \text{ cups} = 32 \text{ cups}$$

Don't worry if you don't know all the measurement conversions—these will be provided on Test Day within the context of the question.

### *Ratios and Proportions*

You most likely first encountered ratios and proportions before you even got to high school, and chances are that you will see them again in college math courses. You will certainly see them on the SAT, so be sure to spend adequate time making sure you have a good grasp of the material in this practice set.

A *ratio* is a relationship that compares the relative size of two amounts. You might see ratios written with a colon, 2:5, as a fraction,  $\frac{2}{5}$ , or using words, 2 to 5. Ratios can compare parts to parts or parts to wholes. For example, suppose you make a fruit salad using 6 oranges, 3 apples, and 2 pears. The ratio of apples to pears is 3:2. The ratio of apples to all the fruit is 3:11.

It is also possible to combine ratios. If you have two ratios,  $a:b$  and  $b:c$ , you can derive  $a:c$  by finding a common multiple of the  $b$  terms. For example, suppose the ratio of  $a$  to  $b$  is 3:4 and the ratio of  $b$  to  $c$  is 5:2. Take a look at the following chart to see how to find the ratio of  $a$  to  $c$ .

$a$	:	$b$	:	$c$
3	:	4		
		5	:	2
15	:	20		
		20	:	8
15			:	8

The number 20 is the least common multiple of 4 and 5, so write 20 in the middle column under the  $b$  terms. Next, multiply each ratio by the factor (use 5 for  $a:b$  and 4 for  $b:c$ ) that gives  $b = 20$ . Finally, write the resulting relationship between  $a$  and  $c$ : The ratio of  $a$  to  $c$  is 15:8.

*Proportions* are simply two ratios set equal to each other. They are an efficient way to solve certain problems, but you must exercise caution when setting them up. Watching the units of each piece of the proportion is critical. Writing the proportion in words first is a good way to avoid careless errors. To solve a proportion, cross-multiply and then use inverse operations to isolate the unknown quantity. You can also use cross-multiplication to verify that two ratios are proportional. For example

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } ad = bc.$$

### Percentages

A percentage is a type of proportion that means “per 100.” Not only are percentages a common occurrence on the SAT, they’re also common in daily life. These two reasons alone are enough for you to know that percentages are a concept you’re going to need to understand.

Here are some useful formulas to learn before Test Day:

- Percent  $\times$  whole = part
- Percent =  $\frac{\text{part}}{\text{whole}} \times 100\%$
- Percent change =  $\frac{\text{amount of change}}{\text{initial amount}} \times 100\%$

Keep in mind that when using percentages in calculations, you must write the percent as a decimal number.

## PRACTICE SET



You may use your calculator for all questions in this practice set.

### Easy

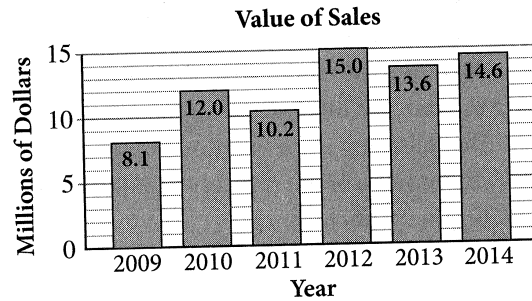
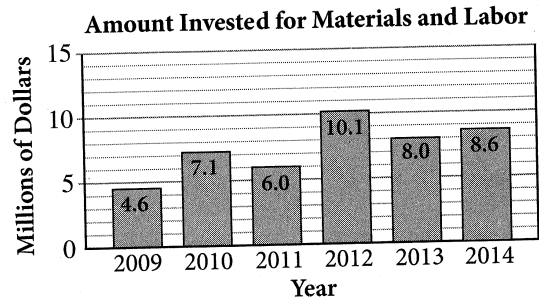
- An architect is working on a design for a solar power plant. While double-checking the blueprint, he realizes that he has made an error: The solar panels will not fit on the roof unless the dimensions of the roof are increased by 0.5%. Which of the following expresses this change in the dimensions of the roof as a fraction of the original dimensions?
  - $\frac{1}{2,000}$
  - $\frac{1}{200}$
  - $\frac{1}{20}$
  - $\frac{1}{2}$
- A graphic designer created an image for a school banner. He uses this image for small and medium banners. To produce a large banner, the designer increases the size of the image by 112.5% to 18 inches tall by 22.5 inches wide. What was the size of the original image?
  - 15 inches by 17.5 inches
  - 15 inches by 18 inches
  - 16 inches by 18.5 inches
  - 16 inches by 20 inches
- A typical song downloaded from the Internet is 4 megabytes in size. Lindy has satellite Internet, and her computer downloads music at a rate of 256 kilobytes per second. If 1 megabyte equals 1,024 kilobytes, about how many songs can Lindy download in 2 hours?
  - 128
  - 450
  - 1,800
  - 1,920
- A subway car passes 3 stations every 10 minutes. At this rate, how many stations will it pass in 1 hour?
 

	7	7		
	0	0	0	
1	1	1	1	
2	2	2	2	
3	3	3	3	
4	4	4	4	
5	5	5	5	
6	6	6	6	
7	7	7	7	
8	8	8	8	
9	9	9	9	

**Medium**

5. Cecilia leaves home for school on her bike at 9:05 AM, riding at an average speed of 10 miles per hour. Fifteen minutes later, her mother realizes that Cecilia forgot to take her lunch. Cecilia's mother immediately gets into her car and drives after Cecilia at an average speed of 25 miles per hour. At what time will Cecilia's mom catch up to her?

- A) 9:15 AM
- B) 9:20 AM
- C) 9:26 AM
- D) 9:30 AM



6. The bar graphs show the amounts, in dollars, invested by a company for materials and labor to produce a certain product and the value of the sales for that product. The company claims that the amount invested for materials and labor is never more than 60% of the value of the sales. Which year or years disprove this claim?
- A) 2010
  - B) 2012
  - C) 2014
  - D) 2012 and 2014

7. A sporting goods store ordered an equal number of white and yellow tennis balls. The tennis ball company delivered 30 extra white balls, making the ratio of white balls to yellow balls 6:5. How many tennis balls did the store originally order?
- A) 120  
 B) 150  
 C) 240  
 D) 300

Metal	Density (g/cm <sup>3</sup> )
Copper	8.96
Iron	7.87
Nickel	8.90
Tin	7.26

8. The *density* of a substance can be found by dividing the mass of the substance by the volume of the substance. The table gives the density of several pure metals in grams per cubic centimeter. Suppose a rectangular sheet of a pure metal weighs about 515.9 grams and measures  $\frac{1}{4}$  inch by 2 inches by 8 inches. Assuming the sheet is one of the metals in the table, which metal is it? (There are approximately 2.54 centimeters in 1 inch.)
- A) Copper  
 B) Iron  
 C) Nickel  
 D) Tin

9. A bike messenger delivers a package from the courthouse to a law firm downtown, traveling at an average speed of 24 miles per hour. On the way back, the bike messenger gets stuck in traffic, which reduces his average speed for the return trip by 12.5 percent. What was the bike messenger's average speed, in miles per hour, for the round trip?
- A) 22  
 B) 22.1  
 C) 22.4  
 D) 22.5

10. The mayor of Dunderville proposes a new ordinance that would give a tax credit to parents who homeschool their children. Ten percent of Dunderville's residents approve of the ordinance. Of these, 7 out of 10 are women. All of Dunderville's other residents are against the ordinance. If there are equal numbers of men and women in Dunderville, then what fraction of Dunderville's male residents approve of the ordinance?
- A)  $\frac{3}{10}$   
 B)  $\frac{3}{47}$   
 C)  $\frac{3}{50}$   
 D)  $\frac{3}{100}$

11. Each crop duster who works for Gary's Crop Dustin' 4 Cheap can dust half an acre of crops in 25 minutes. The company must dust eight 0.75-acre lots and twelve 1.5-acre lots to complete a certain job. What is the minimum number of crop dusters needed to complete the job in 5 hours?
- A) 4  
B) 5  
C) 6  
D) 7
12. Comix Fanatix has 250 customers who subscribe to its newsletter, 68% of whom are male. After some female customers cancel their subscriptions, the total number of subscribers becomes 1.3 times the number of male subscribers. How many female customers canceled their subscriptions?

	7	7	
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13. In preparation for buying a house, Sergey and Juanita draft a budget to see how much they can afford. Their monthly spending and income are shown in the table. Numbers in parentheses indicate expenses (money being spent); numbers without parentheses indicate income (money being earned). Sergey and Juanita will not buy a house unless they can save at least \$400 a month after all expenses, including the new mortgage payment and property taxes, are paid.

Category	Monthly Amount
Sergey's income (after taxes)	\$1,300
Juanita's income (after taxes)	\$1,600
Car insurance and gas	(\$220)
Food	(\$400)
Entertainment	(\$100)
Student loans	(\$550)
Phone and Internet	(\$120)

The couple would like to buy a house that costs \$230,400, for which the monthly mortgage payment would be \$730, and the annual property taxes would be 2.5% of the purchase price. By what percent could Sergey and Juanita cut their monthly food spending in order to buy this particular house and meet their criteria for purchasing a house? (Ignore the percent sign and grid in your answer as a whole number.)

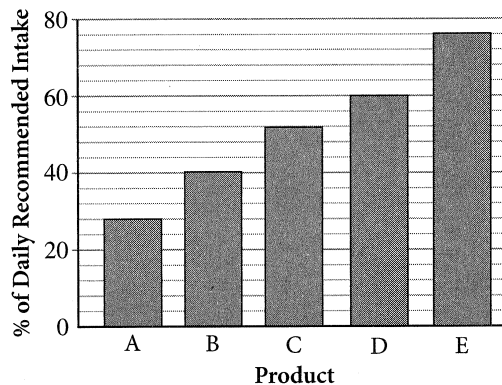
	7	7	
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

**Hard**

14. Amal, Geoffrey, and Julia each have some fireworks. If Amal gives half of his fireworks to Geoffrey, then Amal and Geoffrey will have fireworks in a 2:3 ratio. If Julia has one-fifth as many fireworks as Geoffrey (after Amal gives half his fireworks to Geoffrey), then what is the ratio of the number of fireworks Amal has to the number of fireworks Julia has?
- A) 20:1  
 B) 15:4  
 C) 3:2  
 D) There is insufficient information to determine the answer.
15. An oil tank has two pipes connected to it. If the tank is empty, Pipe A can fill it in 2 hours. If the tank is full, Pipe B can empty it in 3 hours. If both pipes are activated at the same time when the tank is empty, how many hours will it take for the tank to be filled to 60% of its capacity?
- A) 2.4  
 B) 3.6  
 C) 6  
 D) 60
16. An all-natural health clinic stocks teas and herbs in a 5:11 ratio at one of its warehouses. In an effort to cut overhead costs, the owners close this warehouse and transfer its stock to a larger warehouse, increasing that warehouse's inventory of items by 20%. Which of the following could be the number of items at the larger warehouse after the transfer?
- A) 198  
 B) 386  
 C) 480  
 D) 573

17. John buys  $c$  pounds of cheese to feed  $m$  people at a party. If  $m + n$  people come to the party, how many more pounds of cheese must John buy in order to feed everyone at the original rate?
- A)  $\frac{mn}{c}$   
 B)  $\frac{m}{cn}$   
 C)  $\frac{m + n}{c}$   
 D)  $\frac{nc}{m}$

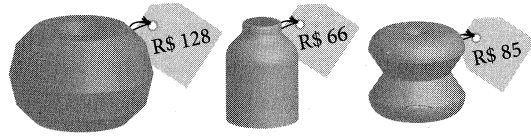
**Percent Calcium in Different Multivitamins**



18. The bar graph above shows the percent of the daily recommended intake of calcium supplied by five different multivitamins, A, B, C, D, and E, per pill. Products A and D are sold in bottles of 75 pills and cost \$12 and \$18, respectively. Products B, C, and E are sold in bottles of 50 pills and cost \$9.50, \$10.25, and \$15.50, respectively. Which of the five products supplies the greatest percentage of the daily-recommended intake of calcium per dollar?
- A) A  
 B) B  
 C) C  
 D) E

19. A typical 3-D printer creates objects by laying down layers of ink one on top of another. The average thickness of a layer depends on the caliber of the printer. For one particular 3-D printer, the average thickness of a layer is  $102\ \mu\text{m}$  (micrometers). When creating a 1-inch cube, the printer takes 6 seconds to lay down each layer of ink. Approximately how long should it take this printer to create a 1-inch cube? (There are 1,000,000 micrometers in a meter, and 1 inch is approximately equal to 0.0254 meters.)
- A) 12 minutes  
 B) 25 minutes  
 C) 40 minutes  
 D) 1 hour, 6 minutes

20. Eli left his home in New York and traveled to Brazil on business. Before he left, he used his credit card to purchase these pewter vases:



For daily purchases totaling less than 200 U.S. dollars, Eli's credit card company charges a 2% fee. If the total charge on his credit card for the vases was \$126.48, what was the foreign exchange rate in Brazilian reais (R\$) per U.S. dollar on the day that Eli bought the vases? If necessary, round your answer to the nearest hundredth.

