PRACTICE SET 3: INEQUALITIES

Solving Inequalities

Working with inequalities is similar to working with equations, but with a few key differences.

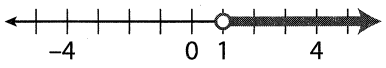
- The language used to describe inequalities tends to be more complex than the language used to describe equations. You "solve" an equation for x, but with an inequality, you might be asked to "describe all possible values of x" or provide an answer that "includes the entire set of solutions for x." This difference in wording exists because an equation describes a specific value of a variable, whereas an inequality describes a range of values.
- Instead of an equal sign, you'll see a sign denoting either "greater/less than" ($>$ and $<$) or "greater/less than or equal to" ($\geq$ and $\leq$). The open end of the inequality symbol should always point toward the greater quantity.
- When solving an inequality that involves multiplying or dividing by a negative number, the inequality symbol must be reversed. For example, if given $-4x < 12$, you must reverse the symbol when dividing by -4, which will yield $x > -3$ (NOT $x < -3$).

Compound Inequalities

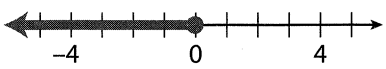
Sometimes you'll see a variable expression wedged between two quantities. This is called a compound inequality. For example, $-5 < 2x + 1 < 11$ is a compound inequality. You solve it the same way (using inverse operations), keeping in mind that whatever you do to one piece, you must do to all three pieces. And, of course, if you multiply or divide by a negative number, you must reverse both inequality symbols.

Graphing Inequalities

Inequalities can be presented graphically in one or two dimensions. In one dimension, inequalities are graphed on a number line with a shaded region. For example, $x > 1$ is graphed like this:

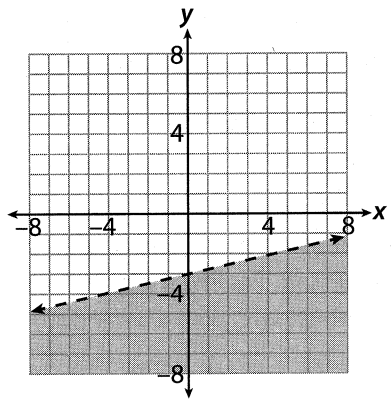


Notice the open dot at 1, which indicates that 1 is not a solution to the inequality. This is called a strict inequality. By contrast, the graph of $x \leq 0$ looks like this:



Notice the solid dot at 0, which indicates that 0 should be included in the solution set for the inequality.

In two dimensions, things get a bit more complicated. While linear equations graph as simple lines, *inequalities* graph as lines called *boundary lines* with shaded regions known as *half planes*. Solid lines indicate inequalities that have ≤ or ≥ because the values on the line itself are included in the solution set. Dashed lines involve strict inequalities that have > or < because, in these cases, the values on the line itself are not included in the solution set. The shaded region (and the line if it is solid) represents all points that make up the solution set for the inequality. For example, the graph below represents the solution set to the inequality $y < \frac{1}{4}x - 3$.



If you're not sure whether to shade above or below a boundary line, plug a pair of coordinates into the inequality. If the coordinates satisfy the inequality, then the region in which that point lies should be shaded. If the coordinates violate the inequality, then that region should not be shaded. An easy point to use when testing inequalities is the origin, (0, 0). In the example above $0 \not< -3$, so the half plane that contains (0, 0) should not be shaded (which it isn't).

Solving Systems of Inequalities

Multiple inequalities can be combined to create a system of inequalities. Systems of inequalities can also be represented graphically with multiple boundary lines and multiple shaded regions. Follow the same rules for graphing single inequalities, but keep in mind that the solution set to the system is the region where the shaded half planes overlap. Shading in different directions (e.g., parallel lines slanted up for one inequality and down for the other) makes the overlap easier to see. Just as with single inequalities, you can pick coordinates to plug into a system of inequalities to determine which side of the boundary lines should be shaded.

PRACTICE SET

Easy

1. Which of the following gives all values of j that satisfy the inequality $3j - 4 \leq 6j + 11$?

   A) $j \leq -5$

   B) $j \geq -5$

   C) $j \leq 5$

   D) $j \geq 5$

2. Which of the following numbers is not a solution to the inequality $6x - 9 \geq 7x - 5$?

   A) -8

   B) -5

   C) -4

   D) -2

3. To take the neighbor's children to the movies, Mellie charges \$5 for gas and \$8 per hour spent with the children. Ron charges \$3 for gas and \$8.50 per hour spent with the children. If h represents the number of hours spent with the children, what are all the values of h for which Ron's total charge is greater than Mellie's total charge?

   A) $h < 3$

   B) $3 < h \leq 4$

   C) $4 \leq h < 5$

   D) $h > 4$

4. Yasmine is a pharmaceutical sales representative. Her firm gives her a weekly allowance of \$300 to spend on lunches with physicians and their office staffs. A restaurant from which Yasmine often buys the lunches charges \$7 for a cold dish and \$11 for a hot dish, including drinks. If each meal is subject to a 5.75% sales tax, which of the following inequalities represents the number of cold dishes (c) and hot dishes (h) that Yasmine can purchase for sales-related lunches in one week, assuming she purchases all the lunches from this restaurant?

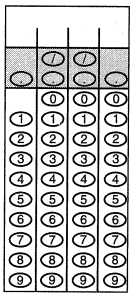
   A) $7c + 11h \leq 1.0575(300)$

   B) $7c + 11h \geq 1.0575(300)$

   C) $1.0575(7c + 11h) \leq 300$

   D) $1.0575(7c + 11h) \geq 300$

5. If $\frac{5}{6} < \frac{1}{2}x - \frac{1}{2}y < \frac{3}{2}$, then what is one possible value of $x - y$?



Medium

6. What is the least possible integer value for which 40% of that integer is greater than 9.6?

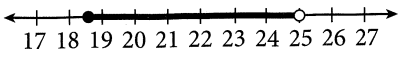
   A) 4

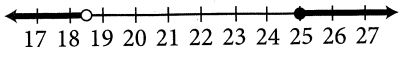
   B) 12

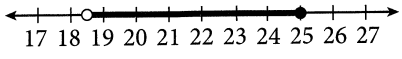
   C) 20

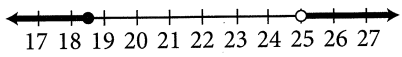
   D) 25

7. Body mass index (BMI) is a comparison of a person's body mass to his or her height. A high BMI can be an indicator of high body fat, which can lead to health problems. According to the American Heart Association, an adult is underweight if his or her BMI is less than 18.5, or overweight if it is greater than or equal to 25.0. Which of the following number lines could be used to model a healthy BMI range for an adult?

A) 

B) 

C) 

D) 

$$\frac{2(4k+1)}{3} \geq \frac{k(6+5)-3}{2}$$

8. Which of the following correctly describes the possible values of k in the inequality above?

A) $k \leq -1$

B) $k \geq -1$

C) $k \leq \frac{13}{17}$

D) $k \geq \frac{13}{17}$

9. A math teacher decides to create several practice tests for her students before they take the SAT. She wants to make some non-calculator tests and some calculator tests so that her students will be able to practice both. She figures that each non-calculator test will take her 3 hours to create, and each calculator test will take 4 hours to create. If she is willing to devote at most 6 hours per week of her time for the next 5 weeks to create the practice tests, and she wants to provide at least 8 practice tests, which of the following systems of inequalities can help her determine how many of each type of test she can create?

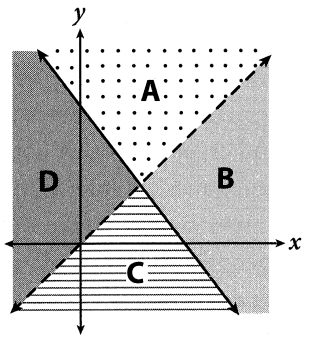
A) $n + c \geq 8$
$3n + 4c \leq 6$

B) $n + c \geq 8$
$3n + 4c \leq 30$

C) $n + c \leq 8$
$3n + 4c \geq 30$

D) $n + c \geq 6$
$3n + 4c \leq 8$

$\begin{cases} y > x \\ 3y \leq -4x + 6 \end{cases}$



10. A system of inequalities and the corresponding graph are shown above. Which part of the graph could represent all of the solutions to the system?

    A) A

    B) B

    C) C

    D) D

11. A housing down payment is money that a prospective buyer provides up front when purchasing a home and is usually a percent of the purchase price of the home. A lender typically requires private mortgage insurance (PMI) when the buyer's down payment is less than 20% of the purchase price. To secure a mortgage, buyers also need to have additional cash on hand for closing costs and prepaid property tax. Suppose a buyer wants to purchase a $375,000 house and must have $7,200 on hand for closing costs and property tax. Which of the following inequalities represents the total funds (f) the buyer must have on hand to secure the mortgage without having to pay PMI?

    A) $f \leq 0.2(375{,}000) + 7{,}200$

    B) $f \geq 0.2(375{,}000) + 7{,}200$

    C) $f \leq 0.2(375{,}000 + 7{,}200)$

    D) $f \geq 0.2(375{,}000 + 7{,}200)$

12. If $-\frac{5}{2} < -2m + 1 < -\frac{7}{5}$, what is the greatest possible integer value of the expression $10m - 5$?

    A) 6

    B) 7

    C) 10

    D) 12

$$18{,}000 + x \leq 72{,}000$$

13. The federal interstate weight limit for a particular four-axle transfer truck is 18,000 pounds per axle. The cab (front) of the truck weighs 11,000 pounds, and the trailer of the truck, when empty, weighs 7,000 pounds. The inequality above represents the legally permissible weight range for this truck when travelling on an interstate. What does the value 18,000 represent in the inequality?

    A) The weight of the truck when fully loaded

    B) The weight of the truck when the trailer is empty

    C) The maximum weight allowed per axle

    D) The maximum weight of the cargo being transported

14. A construction company prepares an estimate to install a new pool for a homeowner. The estimate includes h hours of labor, where $h > 80$. The company's goal is for the estimate to be within 8 hours of the actual number of hours of labor. If the company meets the goal and it takes a hours of actual labor, which inequality represents the relationship between the estimated number of hours of labor and the actual number of hours of labor?

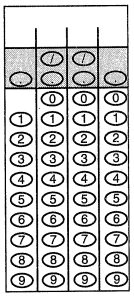
A) $a + h \leq 8$

B) $a \geq h + 8$

C) $a \leq h - 8$

D) $-8 \leq a - h \leq 8$

15. Margo can peel and slice at least 10 dozen apples per hour and at most 15 dozen apples per hour. Based on this information, what is a possible amount of time, in hours, that it could take Margo to peel and slice 60 dozen apples?



Hard

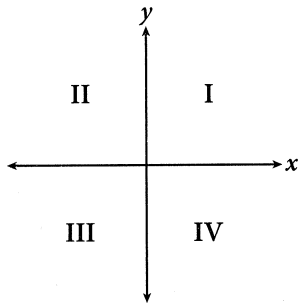
16. The earth is made up of four primary layers: the inner core, outer core, mantle, and crust. The outer core is more than 800 miles and less than 2,200 miles from the Earth's center. Which of the following inequalities represents all possible distances, d, in miles, from the Earth's center that are in the outer core?

A) $|d + 800| < 2{,}200$

B) $|d - 800| < 2{,}200$

C) $|d + 1{,}500| < 700$

D) $|d - 1{,}500| < 700$



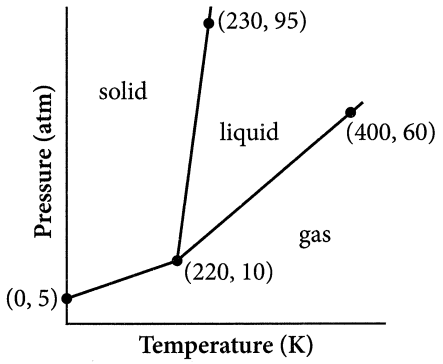
17. If the system of inequalities given by $x + 3y \leq 12$ and $2x - 3y \leq -3$ is graphed on the coordinate plane above, which quadrant of the plane contains no solutions to the system?

A) Quadrant I

B) Quadrant IV

C) Quadrants I and IV

D) Quadrants III and IV



18. A phase diagram shows the temperatures and pressures at which a chemical substance exists in a certain phase (solid, liquid, or gas). A sample phase diagram for a fictional substance is shown in the figure, where T is temperature in Kelvin and P is pressure in atmospheres. Where on the diagram a certain temperature and pressure combination falls determines the state(s) in which the substance exists. For example, if a certain temperature-pressure pairing falls in the gas area (but not on the line segment between gas and liquid), the substance exists only as a gas. However, if the temperature-pressure pairing is on the line segment between gas and liquid, the substance exists as both a gas and a liquid. Assuming $T > 0$ and $P > 0$, which of the following systems of inequalities could be used to describe the temperature and pressure ranges in which this substance exists only as a liquid?

A) $P < \frac{17}{2}T - 1{,}860;\ P > \frac{5}{18}T - \frac{460}{9}$

B) $P > \frac{17}{2}T - 1{,}860;\ P < \frac{5}{18}T - \frac{460}{9}$

C) $P < \frac{17}{2}T - 1{,}860;\ P > \frac{11}{200}T - \frac{460}{9}$

D) $P \geq \frac{17}{2}T - 1{,}860;\ P \leq \frac{11}{200}T - \frac{460}{9}$

$$y < x + k_1$$
$$y > 2x + k_2$$

19. Suppose that on a coordinate plane, (0, 0) is a solution to the system of inequalities given above. Which of the following conclusions about k_1 and k_2 must be true?

A) $k_1 < k_2$

B) $k_2 < k_1$

C) $|k_1| < |k_2|$

D) $k_1 = -k_2$

20. The variables x and y represent numbers for which the statements $x - y > 300$ and $\frac{y}{x} = 0.625$ are true. What is the smallest integer that x can equal?