PRACTICE SET 2: SYSTEMS OF LINEAR EQUATIONS

What Is a System of Equations?

A system of equations is a set of two or more equations that usually contains multiple variables. Systems of equations are extremely useful in modeling and simulation. Complex mathematical problems, such as weather forecasting or crowd control predictions, often require 10 or more equations to be simultaneously solved for multiple variables. Fortunately, virtually all systems of equations on the SAT that require solving will contain only two equations and two variables.

In general, when you are given a situation involving n variables, you need a system of n equations to arrive at fixed values for these variables. Thus, if you have two variables, you need two equations. Three variables would require three equations, and so on.

To solve a two-variable system of equations, the equations that make up the system must be independent. *Independent equations* are equations for which no algebraic manipulations can transform one of the equations into the other.

Consider the equation $4x + 2y = 8$. We could use properties of equality to rewrite this equation in a number of different ways. For example, we could multiply both sides by 2, resulting in the equation $8x + 4y = 16$. While it seems as though we've just created an additional equation, this is misleading, as our second equation has the same core variables and relationships as our first equation. These two equations are called *dependent equations*. Two dependent equations cannot be used to solve for two variables because they really represent the same equation, just written in different forms. If you attempt to solve a system of dependent equations, you will end up with the same thing on both sides of the equal sign (e.g., $16 = 16$), which is always true and indicates that the system has infinitely many solutions.

At other times, you'll encounter equations that are fundamentally incompatible with each other. For example, given the two equations $4x + 2y = 8$ and $4x + 2y = 9$, it should be obvious that there are no values for x and y that will satisfy both equations at the same time. Doing so would violate fundamental laws of math. In this case, the system of equations has no solution.

Solving Systems of Equations

The two main methods for solving a system of linear equations are substitution and combination (sometimes referred to as elimination or elimination by addition).

Substitution is the most straightforward method for solving systems and can be applied in every situation. However, the process can get messy if none of the variables in either of the equations has a coefficient of 1. To use substitution, solve the simpler of the two equations for one variable, and then substitute the result into the other equation. Unfortunately, substitution is often the longer and more time-consuming route for solving systems of equations.

Combination involves adding the two equations, or a multiple of one or both equations, together to eliminate one of the variables. You're left with one equation and one variable, which can be solved using inverse operations.

Caution: Although most students prefer substitution, questions on the SAT that involve systems of equations are often designed to be quickly solved using combination. For instance, combination is almost always the fastest technique to use when solving a system of equations question that only asks for the value of one of the variables. To really boost your score on Test Day, practice combination as much as you can on practice tests and in homework questions so that it becomes second nature.

Graphing Systems of Equations

Knowing how many solutions a system of equations has tells you how graphing the equations on the same coordinate plane should look. Conversely, knowing what the graph of a system of equations looks like tells you how many solutions the system has.

Recall that the solution(s) to a system of equations is the point or points where the graphs of the equations intersect. The table below summarizes three possible scenarios:

If your system has ...	... then it will graph as:	Reasoning
no solution	two parallel lines	Parallel lines never intersect.
one solution	two lines intersecting at a single point	Two straight lines have only one intersection.
infinitely many solutions	a single line (one line directly on top of the other)	One equation is a manipulation of the other—the graphs are the same line.

Translating Word Problems into Multiple Equations

Sometimes questions on the SAT will present information in a real-world context, and you'll need to translate it into a system of equations and then solve. This might sound scary, but solving systems of equations can be relatively straightforward once you get the hang of it. Just remember to use the Kaplan Strategy for Translating English into Math to set up your equations as you did in the previous practice set, and then solve using either substitution or combination.

PRACTICE SET

Easy

$$x + y = 29$$
$$x + 2y = 12$$

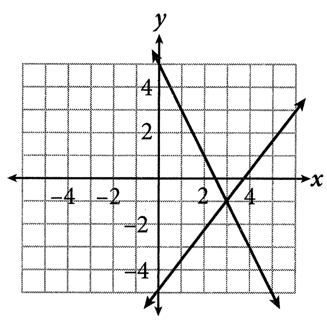
1. If the ordered pair (x, y) is the solution to the system of equations above, what is the value of y?

A) -17

B) 12

C) 46

D) 75



2. The graph of a system of linear equations is shown above. If the ordered pair (x, y) represents the solution to this system, what is the value of $x - y$?

A) -4

B) -2

C) 2

D) 4

3. If $5b = 6a + 16$ and $9a = 7b - 20$, then what is the value of $3a - 2b$?

A) -8

B) -4

C) 4

D) 8

$$4x - 2y + 3 = 8$$
$$3x + 6y = 8y - x + 5$$

4. How many solutions does the system of equations shown above have?

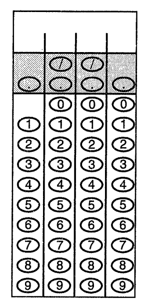
A) 0

B) 1

C) 2

D) Infinitely many

5. An office has 27 employees. If there are seven more women than men in the office, how many employees are men?



Medium

6. The total fare for two adults and three children on an excursion boat is $14. If each child's fare is one-half of each adult's fare, what is the total cost for one adult and one child?

A) $4.00

B) $5.25

C) $6.00

D) $6.50

$$y = \frac{1}{5}x + 4$$
$$y = \frac{3}{7}x - 4$$

7. If the ordered pair (x, y) satisfies the system of equations above, what is the value of y?

   A) 0

   B) 7

   C) 10

   D) 11

$$2x - 3y = -3$$
$$-12 = -4x + y$$

8. In what quadrant will the lines represented by the equations above intersect?

   A) Quadrant I

   B) Quadrant II

   C) Quadrant III

   D) Quadrant IV

9. If $10a = 6b + 7$ and $a - 6b = 34$, then what is the value of $-\frac{1}{3}a$?

   A) -1

   B) 1

   C) $\frac{41}{27}$

   D) $\frac{41}{9}$

10. In addition to the standard airfare, a particular airline charges passengers for two kinds of travel services: \$25 to check a bag and \$15 to upgrade to priority boarding. If the airline collected \$3,065 in baggage and priority boarding fees from 145 travel services on two flights combined, which of the following systems of equations could be used to determine the number of bags checked (b) and the number of priority boarding upgrades (p) purchased on the two flights?

    A) $b + p = 145 \times 2$
    $25b + 15p = 3{,}065 \times 2$

    B) $b + p = 145$
    $25b + 15p = 3{,}065$

    C) $b + p = 145$
    $15b + 25p = 3{,}065$

    D) $b + p = \frac{145}{2}$
    $15b + 25p = \frac{3{,}065}{2}$

11. The most popular items at a bakery are its raspberry scones and its lemon poppy seed muffins. The shop sells both items in boxes of 12 at a cost of \$15 per box of raspberry scones and \$9 per box of lemon poppy seed muffins. On Friday and Saturday, the shop earned \$396 by selling a total of 46 boxes of these two items. If r and l represent the number of boxes of raspberry scones and lemon poppy seed muffins sold over the two-day period, respectively, which of the following systems of equations could be used to find the number of boxes of each type of item sold?

A) $r + l = 46$
$15r + 9l = 396$

B) $r + l = 12 \times 46$
$15r + 9l = 396$

C) $r + l = 46$
$15r + 9l = \frac{396}{2}$

D) $r + l = 46 \times 12$
$15r + 9l = \frac{396}{12}$

12. Tricia manages a health bar and wants to add a new fruit-and-protein smoothie to the menu. To decide on the first new flavor she plans to offer, Tricia sold trial-sized banana smoothies and kiwi smoothies. She charged \$2 for a banana smoothie and \$2.50 for a kiwi smoothie, and she sold 40 in all, totaling \$87. How much more money did Tricia make on the banana smoothies than the kiwi smoothies?

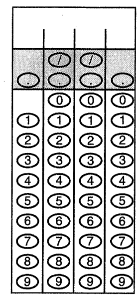
A) \$12

B) \$17

C) \$26

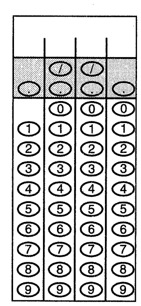
D) \$52

13. A street vendor sells two types of newspapers, one for \$0.25 and the other for \$0.40. If she sold 100 newspapers for \$28.00, how many newspapers did she sell at \$0.25?



$$x + y = -6$$
$$y - 4x = 4$$

14. If the ordered pair (x, y) satisfies the system of equations shown above, what is the value of xy?



Hard

$$4x + 7y = 24$$
$$6x + \frac{21}{2}y = g$$

15. In the system of equations above, g is a constant. If the system has infinitely many solutions, what is the value of g?

A) 16

B) 32

C) 36

D) 72

16. A small office supply store sells paper clips in packs of 100 and packs of 250. If the store has 84 packs of paper clips in stock totaling 12,300 paper clips, how many paper clips would a customer buy if he buys half of the packs of 250 that the store has in stock?

A) 2,900

B) 3,250

C) 5,800

D) 6,500

17. In a college art class, 76 students are painting a mural on one wall of the campus amphitheater. The wall has been divided into 23 sections, and each section will be painted by a group of either 2 or 4 students. How many more sections will be painted by a group of 4 students than by a group of 2 students?

A) 6

B) 7

C) 8

D) 9

$$\frac{5}{8}x + \frac{7}{2}y = \frac{3}{2}$$

$$\frac{1}{6}x - \frac{2}{3}y = 1$$

18. If the ordered pair (x, y) satisfies the system of equations above, what is the sum of the values of x and y?

A) $\frac{5}{24}$

B) $\frac{5}{2}$

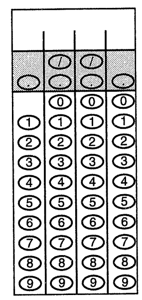
C) $\frac{29}{8}$

D) $\frac{33}{8}$

$$11x - 24y = 8$$

$$kx - 36y = 5$$

19. In the system of equations above, k is a constant. If the system has no solution, what is the value of k?



20. Two moonflower vines are growing on a trellis in Mallory's backyard. She bought the first vine when it was 11 inches long and found that it grows at a rate of 0.25 inches per day. Exactly 20 days later, Mallory bought the second vine, which started at 24 inches long and has a growth rate of 0.125 inches per day. How many days will Mallory have had the first vine when the lengths of the two vines are the same?