

More practices on factoring and rational expressions  
From AMC 10's

1. The ratio  $\frac{10^{2000} + 10^{2002}}{10^{2001} + 10^{2001}}$  is closest to which of the following numbers?

- (A) 0.1    (B) 0.2    (C) 1    (D) 5    (E) 10

$$\frac{10^{2000}(1 + 10^2)}{10^{2000}(10 + 10)} = \frac{101}{20} \approx 5.05$$

1. The ratio  $\frac{2^{2001} \cdot 3^{2003}}{6^{2002}}$  is

- (A)  $\frac{1}{6}$     (B)  $\frac{1}{3}$     (C)  $\frac{1}{2}$     (D)  $\frac{2}{3}$     (E)  $\frac{3}{2}$

$$\frac{2^{2001} \cdot 3^{2003}}{2^{2002} \cdot 3^{2002}} = \frac{3}{2}$$

3. Which of the following is equal to  $1 + \frac{1}{1 + \frac{1}{1+1}}$ ?  $= 1 + \frac{1}{1 + \frac{1}{2}} = 1 + \frac{1}{\frac{3}{2}} = 1 + \frac{2}{3} = \frac{5}{3}$

- (A)  $\frac{5}{4}$     (B)  $\frac{3}{2}$     (C)  $\frac{5}{3}$     (D) 2    (E) 3

4. What is the value of

$$(3x - 2)(4x + 1) - (3x - 2)4x + 1$$

when  $x = 4$ ?  $= (3x - 2)(4x + 1 - 4x) + 1 = (3x - 2) + 1 = 11$

- (A) 0    (B) 1    (C) 10    (D) 11    (E) 12

6. For how many positive integers  $n$  is  $n^2 - 3n + 2$  a prime number?

- (A) none    (B) one    (C) two    (D) more than two, but finitely many  
(E) infinitely many

$$(n-2)(n-1)$$

if  $n=1, n=3 \rightarrow 1 \cdot 2 = 2 \checkmark$   
if  $n=2 \rightarrow 0 \cdot 1 = 0 \times$

$$\sqrt{\frac{(x)x}{(1-\frac{x-1}{x})x}} = \sqrt{\frac{x^2}{x-x+1}} = \sqrt{x^2} = |x| \rightarrow \text{since } x < 0, |x| = -x$$

7. Which of the following is equivalent to  $\sqrt{\frac{x}{1-\frac{x-1}{x}}}$  when  $x < 0$ ?

- (A)  $-x$  (B)  $x$  (C)  $1$  (D)  $\sqrt{\frac{x}{2}}$  (E)  $x\sqrt{-1}$

7. The fraction

$$\frac{(3^{2008})^2 - (3^{2006})^2}{(3^{2007})^2 - (3^{2005})^2} = \frac{(3^{2005})^2((3^3)^2 - (3)^2)}{(3^{2005})^2((3^2)^2 - 1)}$$

simplifies to which of the following?

- (A)  $1$  (B)  $\frac{9}{4}$  (C)  $3$  (D)  $\frac{9}{2}$  (E)  $9$

$$= \frac{3(27+3)(27-3)}{(9+1)(9-1)} = 9$$

10. Compute the sum of all the roots of  $(2x+3)(x-4) + (2x+3)(x-6) = 0$ .

- (A)  $7/2$  (B)  $4$  (C)  $5$  (D)  $7$  (E)  $13$

$$(2x+3)((x-4) + (x-6)) = 0$$

$$(2x+3)(2x-10) = 0 \quad x = -\frac{3}{2}, 5$$

20. Suppose that the number  $a$  satisfies the equation  $4 = a + a^{-1}$ . What is the value of  $a^4 + a^{-4}$ ?

- (A)  $164$  (B)  $172$  (C)  $192$  (D)  $194$  (E)  $212$

$$(a + \frac{1}{a})^2 = 4$$

$$a^2 + 2 + \frac{1}{a^2} = 16$$

$$a^2 + \frac{1}{a^2} = 14$$

$$(a^2 + \frac{1}{a^2})^2 = 14^2$$

$$a^4 + 2 + \frac{1}{a^4} = 196$$

$$a^4 + \frac{1}{a^4} = 194$$

$$5 + (-\frac{3}{2}) = \frac{7}{2}$$