

$$\begin{aligned} & \rightarrow 1000000 - 4 \\ & = (1000 + 2)(1000 - 2) \\ & = 1002 \cdot 998 \rightarrow 1002 + 998 \\ & = 2000 \end{aligned}$$

Factoring
Pre-Calculus RH

From SAT Math Subj

1. What is the value of x^2 if $x = \sqrt{15^2 - 12^2}$?
a) $\sqrt{3}$ b) 3 c) 9 d) 81 e) 81^2

$$\begin{aligned} & \sqrt{(15+12)(15-12)} \\ & = \sqrt{27 \cdot 3} = 9 \end{aligned}$$

2. Let a be a nonzero constant. If $2x^2 - 4 = a$, then $x^2 - 2 =$
a) $\frac{1}{2}$ b) $\frac{a}{2}$ c) $\frac{2}{a}$ d) 2 e) $2a$

$$\begin{aligned} & \frac{2x^2 - 4}{2} = \frac{a}{2} \\ & x^2 - 2 = \frac{a}{2} \end{aligned}$$

From math competitions,

3. Suppose that b and c are constants and $(x+2)(x+b) = x^2 + cx + 6$. What is c ?
a) -5 b) -3 c) -1 d) 3 e) 5

$$3x + 2x = 5x \rightarrow c = 5$$

4. Suppose that 1998 is written as a product of two positive integers whose difference is as small as possible. What is the difference?
a) 8 b) 15 c) 17 d) 47 e) 93

$$\begin{aligned} & 2000 - 2 \\ & 2(1000 - 1) \\ & 2(10 - 1)(10^2 + 10 + 1) \\ & 2 \cdot 9 \cdot 111 \\ & = \frac{2 \cdot 3 \cdot 3 \cdot 3 \cdot 37}{54} \cdot 37 \rightarrow 54 \cdot 37 \rightarrow 54 - 37 = 17 \end{aligned}$$

5. Suppose that a and b are integers such that $x^2 - x - 1$ is a factor of $ax^3 + bx^2 + 1$. What is b ?
a) -2 b) -1 c) 0 d) 1 e) 2

$$(x^2 - x - 1)(ax + c) = ax^3 + bx^2 + 1$$

$$\begin{aligned} & (-1)c = 1 \rightarrow c = -1, \quad -x(-1) - 1(ax) = 0 \\ & \quad \quad \quad x - ax = 0 \rightarrow a = 1 \\ & x^2(-1) - x(1x) = bx^2 \\ & -2x^2 = bx^2 \quad b = -2 \end{aligned}$$

6. The product of four positive integers $a, b, c,$ and d is $8!$, and they satisfy the equations

$$\begin{aligned} ab + a + b &= 524 \\ bc + b + c &= 146 \\ cd + c + d &= 104 \end{aligned}$$

- What is $a - d$?
a) 4 b) 6 c) 8 d) 10 e) 12

7. What is the smallest possible sum of two positive integers whose product is 999,996?

$$\begin{aligned} b) \quad & (a+1)(b+1) = 525 = 3 \cdot 5^2 \cdot 7 \\ & (b+1)(c+1) = 147 = 3 \cdot 7^2 \\ & (c+1)(d+1) = 105 = 3 \cdot 5 \cdot 7 \end{aligned}$$

$$\begin{aligned} & \text{If } b+1 = 21, \quad \left. \begin{array}{l} a+1 = 25 \\ c+1 = 7 \\ d+1 = 15 \end{array} \right\} \begin{array}{l} b = 20 \\ a = 24 \\ c = 6 \\ d = 14 \end{array} \end{aligned}$$

$$\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{20 \cdot 24 \cdot 6 \cdot 14}$$

$$a - d = 10$$