Polynomials
Pre-Calculus RH

Exercise 1 Let $P(x)$ be a linear polynomial with $P(6) - P(2) = 12$. What is $P(12) - P(2)$?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

Exercise 2 Let $x_1 \neq x_2$ be such that $3x_1^2 - hx_1 = b$ and $3x_2^2 - hx_2 = b$. What is $x_1 + x_2$?

(A) $-\frac{h}{3}$ **(B)** $\frac{h}{3}$ **(C)** $\frac{b}{3}$ **(D)** $2b$ **(E)** $-\frac{b}{3}$

Exercise 3 What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

(A) 0 **(B)** 1 **(C)** 49 **(D)** 50 **(E)** 51

Exercise 4 What is the maximum number of points of intersection of the graphs of two different fourth-degree polynomial functions $y = P(x)$ and $y = Q(x)$, each with leading coefficient 1?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

Exercise 5 The parabola with equation $y(x) = ax^2 + bx + c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $y_r(x) = dx^2 + ex + f$. Which of the following equals $a + b + c + d + e + f$?

(A) $2b$ **(B)** $2c$ **(C)** $2a + 2b$ **(D)** $2h$ **(E)** $2k$

Exercise 6 Let $P(x)$ be a polynomial which when divided by $x - 19$ has the remainder 99, and when divided by $x - 99$ has the remainder 19. What is the remainder when $P(x)$ is divided by $(x - 19)(x - 99)$?

(A) $-x + 80$ **(B)** $x + 80$ **(C)** $-x + 118$ **(D)** $x + 118$ **(E)** 0

Exercise 7 The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the average of its zeros, the product of its zeros, and the sum of its coefficients are all equal. The y-intercept of the graph of $y = P(x)$ is 2. What is b?

(A) -11 **(B)** -10 **(C)** -9 **(D)** 1 **(E)** 5

Exercise 8 Suppose that $P(x/3) = x^2 + x + 1$. What is the sum of all values of x for which $P(3x) = 7$?

(A) $-\frac{1}{3}$ **(B)** $-\frac{1}{9}$ **(C)** 0 **(D)** $\frac{5}{9}$ **(E)** $\frac{5}{3}$

Exercise 9 For how many values of the coefficient a do the equations

$$0 = x^2 + ax + 1 \quad \text{and} \quad 0 = x^2 - x - a$$

have a common real solution?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

Exercise 10 The solutions of the equation $x^2 + px + q = 0$ are the cubes of the solutions of the equation $x^2 + mx + n = 0$. Which of the following must be true?

(A) $p = m^3 + 3mn$ **(B)** $p = m^3 - 3mn$ **(C)** $p = 3mn - m^3$

(D) $p + q = m^3$ **(E)** $\left(\frac{m}{n}\right)^3 = \frac{p}{q}$