Logs
Pre-Calc RH

Competition level

Exercise 1 Suppose that

$$\log_2(\log_3(\log_5(\log_7 N))) = 11.$$

How many different prime numbers are factors of N?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 7

Exercise 2 Suppose that

$$\log_2(\log_2(\log_2 x)) = 2.$$

How many digits are in the base-10 representation for x?

(A) 5 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 13

Exercise 3 How many positive integers b have the property that $\log_b 729$ is also a positive integer?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Exercise 4 Let $f(n) = \log_{2002} n^2$ for all positive integers n. Define

$$N = f(11) + f(13) + f(14).$$

Which of the following relations is true?

(A) $N > 1$ **(B)** $N = 1$ **(C)** $1 < N < 2$ **(D)** $N = 2$
(E) $N > 2$

Exercise 5 For some real numbers a and b, the equation

$$8x^3 + 4ax^2 + 2bx + a = 0$$

has three distinct positive roots, and the sum of the base-2 logarithms of the roots is 5. What is the value of a?

(A) -256 **(B)** -64 **(C)** -8 **(D)** 64 **(E)** 256

Exercise 6 For any positive integer n, define

$$f(n) = \begin{cases} \log_8 n, & \text{if } \log_8 n \text{ is rational,} \\ 0, & \text{otherwise.} \end{cases}$$

What is $\sum_{n=1}^{1997} f(n)$?

(A) $\log_8 2047$ **(B)** 6 **(C)** $\frac{55}{3}$ **(D)** $\frac{58}{3}$ **(E)** 585

Exercise 7 What is the value of the expression

$$N = \frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \cdots + \frac{1}{\log_{100} 100!}\,?$$

(A) 0.01 **(B)** 0.1 **(C)** 1 **(D)** 2 **(E)** 10

Exercise 8 The graph, G, of $y = \log_{10} x$ is rotated 90° counter-clockwise about the origin to obtain a new graph, G'. Which of the following is an equation for G'?

(A) $y = \log_{10}\left(\frac{x+90}{9}\right)$ **(B)** $y = \log_x 10$ **(C)** $y = \frac{1}{x+1}$

(D) $y = 10^{-x}$ **(E)** $y = 10^x$

Exercise 9 What is the value of the sum

$$S = \log_{10}(\tan 1^\circ) + \log_{10}(\tan 2^\circ) + \cdots + \log_{10}(\tan 88^\circ) + \log_{10}(\tan 89^\circ)\,?$$

(A) 0 **(B)** $\frac{1}{2}\log_{10}\left(\frac{1}{2}\sqrt{3}\right)$ **(C)** $\frac{1}{2}\log_{10} 2$ **(D)** $\frac{1}{2}\log_{10} 3$ **(E)** 1

Exercise 10 Let $a \geq b > 1$. What is the largest possible value of

$$\log_a \frac{a}{b} + \log_b \frac{b}{a}\,?$$

(A) -2 **(B)** 0 **(C)** 2 **(D)** 3 **(E)** 4