Functions
Pre Calc RH

From SAT II,

5. If $f(x) = 3\ln(x) - 1$ and $g(x) = e^x$, then $f(g(5)) =$

(A) 6.83
(B) 12
(C) 14
(D) 45.98
(E) 568.17

7. If $f(x) = \sqrt{0.3x^2 - x}$ and $g(x) = \frac{x+1}{x-1}$, then $g(f(10)) =$

(A) 0.2 (B) 1.2 (C) 1.6 (D) 4.5 (E) 5.5

10. If $f(g(x)) = \frac{2\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1}$ and $f(x) = \frac{2x-1}{x+1}$, then $g(x) =$

(A) $\sqrt{x}$
(B) $\sqrt{x^2+1}$
(C) x
(D) x^2
(E) $x^2 + 1$

14. If $f(x) = 3x + 5$ and $f(g(1)) = 11$, which of the following could be $g(x)$?

(A) $7x - 5$
(B) $5x + 7$
(C) $5x - 7$
(D) $5x + 3$
(E) $-5x + 3$

20. If a and b are in the domain of a function f and $f(a) < f(b)$, which of the following must be true?

(A) $a = 0$ or $b = 0$
(B) $a < b$
(C) $a > b$
(D) $a \neq b$
(E) $a = b$

24. What is the domain of $f(x) = \sqrt[3]{-x^2 + 13}$?

(A) $x > 0$
(B) $x > 2.35$
(C) $-2.35 < x < 2.35$
(D) $-3.61 < x < 3.61$
(E) All real numbers

28. If $f(-x) = f(x)$ for all real numbers x and if $(3, 8)$ is a point on the graph of f, which of the following points must also be on the graph of f ?

(A) $(-8, -3)$
(B) $(-3, -8)$
(C) $(-3, 8)$
(D) $(3, -8)$
(E) $(8, 3)$

29. If $f(2x + 1) = 2x - 1$ for all real numbers x, then $f(x) =$

(A) $-x + 1$
(B) $x - 1$
(C) $x - 2$
(D) $2x - 1$
(E) $\frac{1}{2}x - 1$

31. What is the range of the function defined by

$$f(x) = \begin{cases} x^{\frac{1}{3}}, & x > 2 \\ 2x - 1, & x \leq 2 \end{cases} ?$$

(A) $y > 2^{\frac{1}{3}}$
(B) $y \leq 3$
(C) $2^{\frac{1}{3}} < y < 3$
(D) $y \geq 3$
(E) All real numbers

32. If $f(x) = |5 - 3x|$, then $f(2) =$

(A) $f(-2)$

(B) $f(-1)$

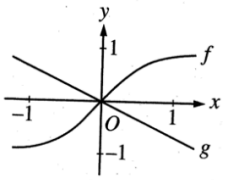
(C) $f(1)$

(D) $f\left(\frac{4}{3}\right)$

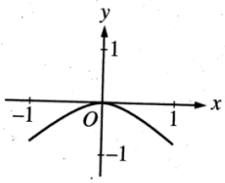
(E) $f\left(\frac{7}{3}\right)$

38. If $f(x) = 5\sqrt{2x}$, what is the value of $f^{-1}(10)$?

(A) 0.04
(B) 0.89
(C) 2.00
(D) 2.23
(E) 22.36



41. Portions of the graphs of f and g are shown above. Which of the following could be a portion of the graph of fg ?

(A)


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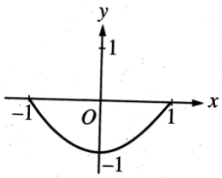

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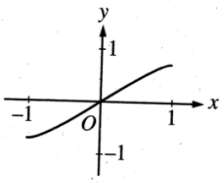

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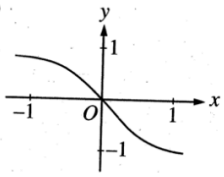
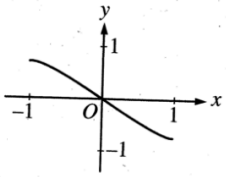


(E)


46. Suppose the graph of $f(x) = -x^2$ is translated 3 units left and 1 unit up. If the resulting graph represents $g(x)$, what is the value of $g(-1.6)$?

(A) 2.96
(B) −0.96
(C) −1.56
(D) −1.96
(E) −2.56

From competitions,

Exercise 1 Suppose that for all $x > 0$ we have $f(2x) = \dfrac{2}{2+x}$. What is $2f(x)$?

(A) $\dfrac{2}{1+x}$ **(B)** $\dfrac{2}{2+x}$ **(C)** $\dfrac{4}{1+x}$ **(D)** $\dfrac{4}{2+x}$ **(E)** $\dfrac{8}{4+x}$

Exercise 2 The function f is defined for positive integers n by:

$$f(n) = \begin{cases} n+3, & \text{if } n \text{ is odd,} \\ n/2, & \text{if } n \text{ is even.} \end{cases}$$

Suppose k is an odd integer and that $f(f(f(k))) = 27$. What is the sum of the digits of k?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 15

Exercise 3 Let $f(x) = ax^7 + bx^3 + cx - 5$, where a, b, and c are constants. Suppose that $f(-7) = 7$. What is $f(7)$?

(A) -17 **(B)** -7 **(C)** 14 **(D)** 17 **(E)** 21

Exercise 4 The function f satisfies $f(2 + x) = f(2 - x)$ for all real numbers x. Moreover, $f(x) = 0$ has exactly four distinct real roots. What is the sum of these roots?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

Exercise 5 Suppose that the function f, for $x \neq -3/2$, is defined by

$$f(x) = \frac{cx}{2x+3},$$

and that $f(f(x)) = x$ for all real numbers in its domain. What is the value of c?

(A) -3 **(B)** $-\dfrac{3}{2}$ **(C)** $\dfrac{3}{2}$ **(D)** 3 **(E)** 5

Exercise 6 Let $f\left(x^2+1\right)=x^4+5x^2+3$. What is $f\left(x^2-1\right)$?

(A) x^4+5x^2+1 **(B)** x^4+x^2-3 **(C)** x^4-3x^2+1
(D) x^4-5x^2+1 **(E)** x^4+x^2+3

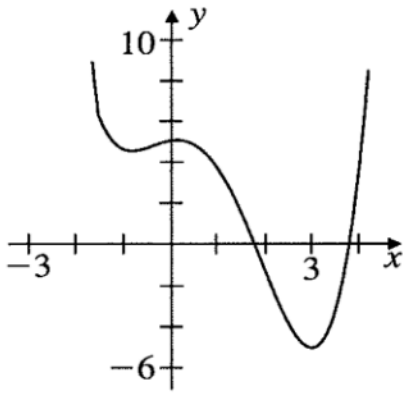
Exercise 7 Suppose that $x^2+y^2=14x+6y+6$. What is the maximum value of $3x+4y$?

(A) 72 **(B)** 73 **(C)** 74 **(D)** 75 **(E)** 76

Exercise 8 What is the number of real solutions of the equation $\frac{x}{100}=\sin x$?

(A) 61 **(B)** 62 **(C)** 63 **(D)** 64 **(E)** 65

Exercise 9 The graph shows a portion of the curve defined by a quartic polynomial of the form $P(x)=x^4+ax^3+bx^2+cx+d$. Which of the following is the smallest?



(A) $P(-1)$ **(B)** The product of the zeros of P.
(C) The product of the non-real zeros of P.
(D) The sum of the coefficients of P.
(E) The sum of the real zeros of P.

Exercise 10 Let $f(x)=x^2+6x+1$, and let R denote the set of points (x, y) in the coordinate plane such that

$$f(x)+f(y)\leq 0 \quad \text{and} \quad f(x)-f(y)\leq 0.$$

Which of the numbers is closest to the area of R?

(A) 21 **(B)** 22 **(C)** 23 **(D)** 24 **(E)** 25