Show your work for full credits. (10 pts each)
1. Simplify.
    a. Eliminate any negative exponent(s). All letters denote positive numbers.

$$\frac{(4ab)^{5/2}}{(8a^3b^{-6})^{2/3}}$$

    b. Rationalize the denominator.

$$\frac{a - \sqrt[3]{2}}{\sqrt[3]{4}}$$

2. Evaluate

$$\sqrt{2001^2 - 1999^2}$$

3. Factor completely, using only positive integer exponents and radicals if necessary.

$$3(2x + 1)^2(2)(x + 1)^{1/2} + (2x + 1)^3 \left(\frac{1}{2}\right)(x + 1)^{-1/2}$$

4. Let $a, b,$ and c be positive integers, where $b > 5$ and satisfy

$$ab + a + b = 17$$
$$bc + b + c = 26$$

What is ac?

5. Simplify the given expression. Use only positive integer exponents and radicals if necessary.

$$\frac{(2 + x^2)^{1/2} - x^2(2 + x^2)^{-1/2}}{2 + x^2}$$

6. The diagonal length of a rectangle is $\sqrt{61}$, and its area is 10. Find its perimeter.

7. Simplify, express without using factorial notation.

$$\frac{(n-2)!}{(n-4)!}$$

8. Let $x + \frac{1}{x} = 3$. What is the value of $x^3 + \frac{1}{x^3}$?

9. Find a value of n, where $n^2 - 8n + 7$ is a prime, where n is a positive integer.

10. Perform the given operation, then simplify. List any restrictions.

i. $\dfrac{4a^2-9}{2a^2+9a-18} \div \dfrac{2a^2+a-3}{a^2+5a-6}$

ii. $\dfrac{a^{-1}+b^{-1}}{(a+b)^{-1}}$