

$$\sin(2A) = 2 \sin A \cos A \quad \leftarrow$$

$$\sin(A+A) = \sin A \cos A + \cos A \sin A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos(A) = 1 - 2\sin^2\left(\frac{A}{2}\right)$$

$$2\sin^2\left(\frac{A}{2}\right) = 1 - \cos A$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$2\theta = A$$

$$\theta = \frac{A}{2}$$

$$\sin(30^\circ) = \frac{1}{2} \quad \sin(15^\circ) = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\sin\left(\frac{30^\circ}{2}\right) = +\sqrt{\frac{1-\cos 30^\circ}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2} \stackrel{?}{=} \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\frac{2\sqrt{2-\sqrt{3}}}{4} \rightarrow 2\sqrt{2-\sqrt{3}} \stackrel{?}{=} \sqrt{6}-\sqrt{2}$$

$$\sqrt{8-4\sqrt{3}} = \sqrt{(a-b)^2}$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$a^2 + b^2 = 8 \rightarrow a^2 + \frac{12}{a^2} = 8$$

$$ab = 2\sqrt{3}$$

$$b = \frac{2\sqrt{3}}{a}$$

$$a^4 + 12 = 8a^2$$

$$a^4 - 8a^2 + 12 = 0$$

$$(a^2 - 6)(a^2 - 2) = 0$$

$$a = \pm\sqrt{6}, \pm\sqrt{2}$$

$$\text{if } a = \sqrt{6}$$

$$b = \frac{2\sqrt{3}}{\sqrt{6}} = \sqrt{2}$$

$$\cos\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\cos A = 2 \cos^2\left(\frac{A}{2}\right) - 1$$

$$\tan\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$= \frac{\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)}$$