33. $\cot(x - y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

$$= \frac{1}{\tan(x-y)}$$

$$= \frac{1+\tan x \tan y}{\tan x - \tan y} \cdot \frac{\frac{1}{\tan x \tan y}}{\frac{1}{\tan x \tan y}}$$

$$= \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

55. Interference Two identical tuning forks are struck, one a fraction of a second after the other. The sounds produced are modeled by $f_1(t) = C \sin \omega t$ and $f_2(t) = C \sin(\omega t + \alpha)$. The two sound waves interfere to produce a single sound modeled by the sum of these functions

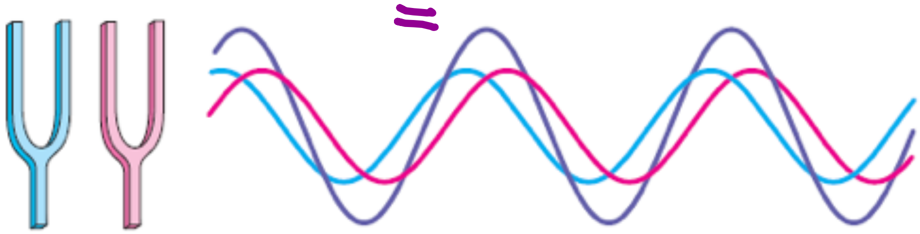
$$f(t) = C \sin \omega t + C \sin(\omega t + \alpha)$$

(a) Use the addition formula for sine to show that f can be written in the form $f(t) = A \sin \omega t + B \cos \omega t$, where A and B are constants that depend on α.

$$f(t) = c \sin wt + c(\sin wt \cos\alpha + \sin\alpha \cos wt)$$

$$= \underbrace{(c + c \cdot \cos\alpha)}_{A} \sin wt + \underbrace{(c \sin\alpha)}_{B} \cos wt$$

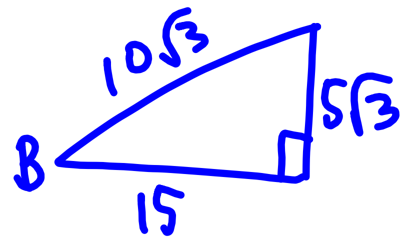
(b) Suppose that $C = 10$ and $\alpha = \pi/3$. Find constants k and ϕ so that $f(t) = k\sin(\omega t + \phi)$.



$$f(t) = (c + c\cdot\cos\alpha)\sin\omega t + (c\sin\alpha)\cos\omega t$$

$$= 10\sqrt{3}\left(\frac{15}{10\sqrt{3}}\sin\omega t + \frac{5\sqrt{3}}{10\sqrt{3}}\cos\omega t\right)$$

$$\sin(A+B) = \cos B\sin A + \sin B\cos A$$



$$B = \cos^{-1}\left(\frac{15}{10\sqrt{3}}\right) = \frac{\pi}{6}$$

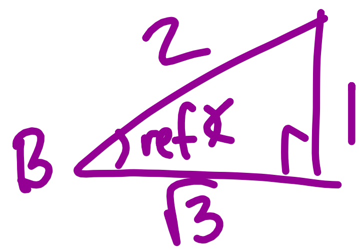
$$10\sqrt{3}\sin\left(\omega t + \frac{\pi}{6}\right) = f(t)$$

$$K = 10\sqrt{3}$$

$$\phi = \frac{\pi}{6}$$

41. $2\left(-\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)$

$\underline{\cos B}\sin A + \underline{\sin B}\cos A$



$= 2\sin\left(x + \frac{5\pi}{6}\right)$

$\theta_r = \frac{\pi}{6}$

$\text{QII}, B = \frac{5\pi}{6}$

$$a\sin x + b\cos x$$

$$\theta_r = \cos^{-1}\frac{|a|}{\sqrt{a^2+b^2}}$$

$$\sqrt{a^2+b^2}\,\sin(x+\theta)$$