31. One method for finding a given number that is in an ordered list of numbers requires a computer to repeatedly split the list in half until the number is found. For a list of n numbers, the maximum number of splits is the least integer greater than or equal to $\frac{\log n}{\log 2}$. What is the maximum number of splits needed to find a given number in a list of 300,000 numbers?

(A) 3 (B) 6 (C) 15 (D) 18 (E) 19

$$\frac{\log 300000}{\log 2}$$

$$= \log_2 300000 = K$$

$$2^n > 2^K = 300000$$

$$2^{10} = 1024$$

$$2^8 = 256$$

$n = 19$

Exercise 10 Let $a \geq b > 1$. What is the largest possible value of

$$\log_a \frac{a}{b} + \log_b \frac{b}{a} ?$$

(A) -2 **(B)** 0 **(C)** 2 **(D)** 3 **(E)** 4

$$\log_a a - \log_a b + \log_b b - \log_b a$$

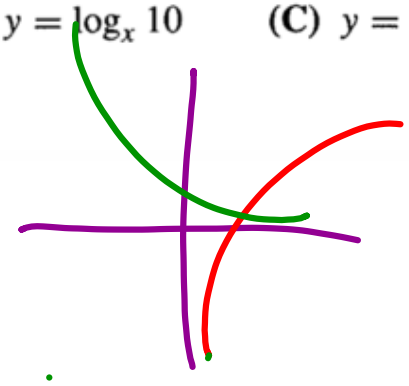
$$= \frac{2}{1} - \frac{x}{1} - \frac{1}{x} = \frac{2x}{x} - \frac{x^2}{x} - \frac{1}{x} = \frac{-x^2+2x-1}{x}$$

$$= \frac{-(x-1)^2}{x} = -\frac{(x^2-2x+1)}{x}$$

Exercise 8 The graph, G, of $y = \log_{10} x$ is rotated 90° counter-clockwise about the origin to obtain a new graph, G'. Which of the following is an equation for G'?

(A) $y = \log_{10}\left(\dfrac{x+90}{9}\right)$ **(B)** $y = \log_x 10$ **(C)** $y = \dfrac{1}{x+1}$

(D) $y = 10^{-x}$ **(E)** $y = 10^{x}$



Exercise 6 For any positive integer n, define

$$f(n) = \begin{cases} \log_8 n, & \text{if } \log_8 n \text{ is rational,} \\ 0, & \text{otherwise.} \end{cases}$$

What is $\displaystyle\sum_{n=1}^{1997} f(n)$?

(A) $\log_8 2047$ **(B)** 6 **(C)** $\dfrac{55}{3}$ **(D)** $\dfrac{58}{3}$ **(E)** 585

51. $\dfrac{1-\cos\alpha}{\sin\alpha} = \dfrac{\sin\alpha}{1+\cos\alpha}$

$$\frac{(1-\cos\alpha)(1+\cos\alpha)}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{1-\cos^2\alpha}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{\sin\alpha\sin\alpha}{\sin\alpha(1+\cos\alpha)} = \frac{\sin\alpha}{1+\cos\alpha}$$