Problem

A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

$\textbf{(A)}\ 202 \qquad \textbf{(B)}\ 223 \qquad \textbf{(C)}\ 224 \qquad \textbf{(D)}\ 225 \qquad \textbf{(E)}\ 234$

$$\begin{array}{r} 223 \ldots R1 \\ 9\overline{)2008} \\ 18 \\ \hline 20 \\ 18 \\ \hline 28 \\ 27 \end{array}$$

$1 \to$

53. $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta\cos^2\theta}{1 \cdot \cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} \cdot \sin^2\theta$$

$$= \tan^2\theta \cdot \sin^2\theta$$

57. $\dfrac{(\sin t + \cos t)^2}{\sin t \cos t} = 2 + \sec t \csc t$

$= \dfrac{\sin^2 t + 2\sin t\cos t + \cos^2 t}{\sin t\cos t}$

$= \dfrac{2\sin t\cos t}{\sin t\cos t} + \dfrac{1}{\sin t\cos t}$

$= 2 + \csc t \cdot \sec t$

65. $\dfrac{\sin x + \cos x}{\sec x + \csc x} = \sin x \cos x$

$\dfrac{\sin x + \cos x}{\frac{1}{\cos x} + \frac{1}{\sin x}} \cdot \dfrac{\sin x \cos x}{\sin x \cos x} = s$

$= \dfrac{(\sin x + \cos x)\sin x \cos x}{\sin x + \cos x} = \sin x \cos x$

49. $2\cos^2 x - 1 = 1 - 2\sin^2 x$

$2(1-\sin^2 x)-1$

$=2-2\sin^2 x-1$

$=1-2\sin^2 x$

61. $\dfrac{\sec x}{\sec x - \tan x} = \sec x\,(\sec x + \tan x)$

$=\dfrac{\sec x\ (\sec x+\tan x)}{(\sec x-\tan x)(\sec x+\tan x)}$

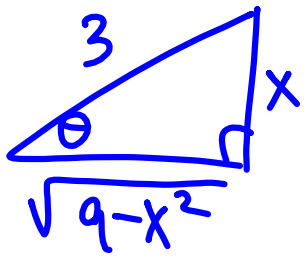
$=\dfrac{\sec x(\sec x+\tan x)}{\sec^2 x-\tan^2 x}$

$=\dfrac{\sec x(\sec x+\tan x)}{1+\tan^2 x-\tan^2 x}=\sec x(\sec x+\tan x)$

93. $\sqrt{9 - x^2}, \quad x = 3 \sin \theta \longrightarrow \sin\theta = \frac{x}{3}$

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$\sqrt{9-x^2} = 3\cos\theta$



73. $\dfrac{\cos\theta}{1 - \sin\theta} = \dfrac{\sin\theta - \csc\theta}{\cos\theta - \cot\theta}$

$= \dfrac{\left(\sin\theta - \frac{1}{\sin\theta}\right)\sin\theta}{\left(\cos\theta - \frac{\cos\theta}{\sin\theta}\right)\sin\theta} = \dfrac{\sin^2\theta - 1}{\sin\theta\cos\theta - \cos\theta}$

$= \dfrac{-\cos\theta\cos\theta}{\cos\theta(\sin\theta - 1)} = \dfrac{\cos\theta}{1 - \sin\theta}$