Problem 5

How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?

(A) 128 (B) 192 (C) 224 (D) 240 (E) 256

$\{2,3,5,7\}$

↳ $2^4 = 16$

$16 - 1 = 15$

$\{4,6,8,9\}$

$4C_0 = 1$

$4C_1 = 4$

$4C_2 = 6$

$4C_3 = 4$

$4C_4 = 1$

16

$2^4 = 16$

$\sum_{i=0}^{n} {}_nC_i = 2^n$

ex) $\{2\} \cup \{4,6\}$

$\{2,4,6\}$

$15 \cdot 16 = 240$

Problem 6

Suppose S cans of soda can be purchased from a vending machine for Q quarters. Which of the following expressions describes the number of cans of soda that can be purchased for D dollars, where 1 dollar is worth 4 quarters?

$\textbf{(A)}\ \frac{4DQ}{S} \quad \textbf{(B)}\ \frac{4DS}{Q} \quad \textbf{(C)}\ \frac{4Q}{DS} \quad \textbf{(D)}\ \frac{DQ}{4S} \quad \textbf{(E)}\ \frac{DS}{4Q}$

P

.25

$SP = \frac{Q}{4}$

$P = \frac{Q}{4S}$

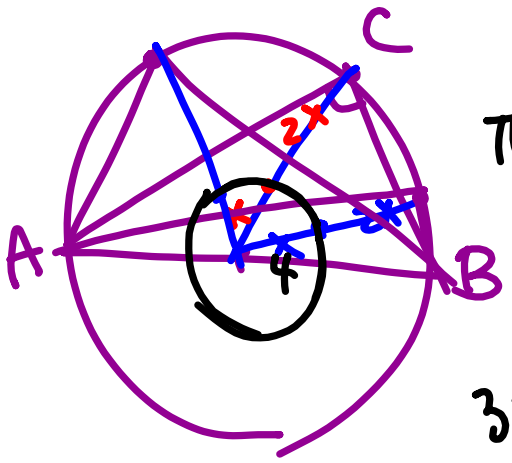
$\frac{D}{P} = \frac{D}{\frac{Q}{4S}} \rightarrow \frac{4DS}{Q}$

Problem 8

Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point C, not equal to A or B, lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

(A) 25 (B) 38 (C) 50 (D) 63 (E) 75

$\frac{352}{7}$



$\pi 4^2$

$= 16\pi$

$\approx 16 \cdot \frac{22}{7}$

$\frac{352}{7}$

Problem 7

What is the value of

$$\log_3 7 \cdot \log_5 9 \cdot \log_7 11 \cdot \log_9 13 \cdots \log_{21} 25 \cdot \log_{23} 27?$$

(A) 3 **(B)** $3\log_7 23$ **(C)** 6 **(D)** 9 **(E)** 10

$$\frac{2\log 25 \cdot \log 27}{\log 3 \log 5} \quad 3$$

Problem 9

What is

$$\sum_{i=1}^{100}\sum_{j=1}^{100}(i+j)?$$

(A) $100, 100$ **(B)** $500, 500$ **(C)** $505, 000$ **(D)** $1, 001, 000$ **(E)** $1, 010, 000$

$100\sum_{i=1}^{100} i$ $\sum_{i=1}^{100} 100i + 5050$ $\sum_{j=1}^{100} j$

$= 505000$ 505000