

### Problem 5

How many subsets of  $\{2, 3, 4, 5, 6, 7, 8, 9\}$  contain at least one prime number?

- (A) 128    (B) 192    (C) 224    (D) 240    (E) 256

$$\{2, 3, 5, 7\}$$

$$\hookrightarrow 2^4 = 16$$

$$16 - 1 = 15$$

ex)  $\{2\} \cup \{4, 6\}$

$$\{2, 4, 6\}$$

$$\{4, 6, 8, 9\}$$

$$4C_0 = 1$$

$$4C_1 = 4$$

$$4C_2 = 6$$

$$4C_3 = 4$$

$$4C_4 = 1$$

$$\begin{array}{r} 1 \\ 4 \\ 6 \\ 4 \\ 1 \\ \hline 16 \end{array}$$

$2^4 = 16$

$$\sum_{i=0}^n {}^n C_i = 2^n$$

$$15 \cdot 16 = 240$$

## Problem 6

→ P

.25

Suppose  $S$  cans of soda can be purchased from a vending machine for  $Q$  quarters. Which of the following expressions describes the number of cans of soda that can be purchased for  $D$  dollars, where 1 dollar is worth 4 quarters?

- (A)  $\frac{4DQ}{S}$    (B)  $\frac{4DS}{Q}$    (C)  $\frac{4Q}{DS}$    (D)  $\frac{DQ}{4S}$    (E)  $\frac{DS}{4Q}$

$$SP = \frac{Q}{4}$$

$$P = \frac{Q}{4S}$$

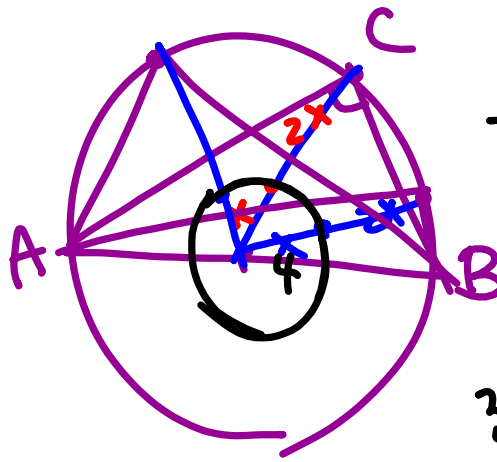
$$\frac{D}{P} = \frac{D}{\frac{Q}{4S}} \Rightarrow \frac{4DS}{Q}$$

**Problem 8**

Line segment  $\overline{AB}$  is a diameter of a circle with  $AB = 24$ . Point  $C$ , not equal to  $A$  or  $B$ , lies on the circle. As point  $C$  moves around the circle, the centroid (center of mass) of  $\triangle ABC$  traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

- (A) 25    (B) 38    (C) 50    (D) 63    (E) 75

$$\frac{352}{7}$$



$$\begin{aligned} \pi 4^2 &= 16\pi \\ &\approx 16 \cdot \frac{22}{7} \\ &= \frac{352}{7} \end{aligned}$$

**Problem 7**

What is the value of

$$\log_3 7 \cdot \log_5 9 \cdot \log_7 11 \cdot \log_9 13 \cdots \log_{21} 25 \cdot \log_{23} 27?$$

- (A) 3    (B)  $3 \log_7 23$     (C) 6    (D) 9    (E) 10

$$\frac{2 \log 25 \cdot \log 27}{\log 3 \cdot \log 5} = 3$$

**Problem 9**

What is

$$\sum_{i=1}^{100} \sum_{j=1}^{100} (i + j)?$$

- (A) 100, 100    (B) 500, 500    (C) 505, 000    (D) 1, 001, 000    (E) 1, 010, 000

Handwritten solution:

$$100 \sum_{i=1}^{100} i$$
$$\sum_{i=1}^{100} 100i + 5050$$
$$= 505000 + 505000$$

The solution shows the original double sum being simplified to  $100 \sum_{i=1}^{100} i$ . This is then expanded to  $\sum_{i=1}^{100} 100i + 5050$ , where the  $5050$  is the sum of  $j$  from  $j=1$  to  $100$ . The final result is  $505000 + 505000 = 1,010,000$ .