Exercise 7 What is the value of the expression

$$N = \frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \cdots + \frac{1}{\log_{100} 100!} ?$$

(A) 0.01 **(B)** 0.1 **(C)** 1 **(D)** 2 **(E)** 10

$$\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$$

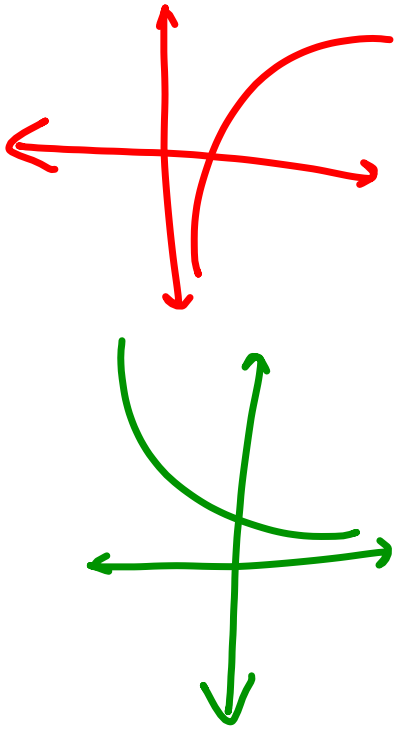
$$\log_{100!} 2 + \log_{100!} 3 \ldots$$

$$\log_{100!} 100! = 1$$

Exercise 8 The graph, G, of $y = \log_{10} x$ is rotated 90° counter-clockwise about the origin to obtain a new graph, G'. Which of the following is an equation for G'?

(A) $y = \log_{10}\left(\frac{x+90}{9}\right)$ **(B)** $y = \log_x 10$ **(C)** $y = \frac{1}{x+1}$

(D) $y = 10^{-x}$ **(E)** $y = 10^x$



$(a,b) \to (-b,a)$

reflect $y=x$

$(x_1,y_1) \to (-y_1, x_1)$

$x_2 \quad y_2$

$y = \log_{10} x$

$y_1 = -x_2$

$x_1 = y_2$

$-x_2 = \log_{10} y_2$

$y_2 = 10^{-x_2}$

Exercise 9 What is the value of the sum

$$S = \log_{10}(\tan 1^\circ) + \log_{10}(\tan 2^\circ) + \cdots + \log_{10}(\tan 88^\circ) + \log_{10}(\tan 89^\circ)\,?$$

(A) 0 **(B)** $\frac{1}{2}\log_{10}\left(\frac{1}{2}\sqrt{3}\right)$ **(C)** $\frac{1}{2}\log_{10} 2$ **(D)** $\frac{1}{2}\log_{10} 3$ **(E)** 1

① $\log_{10} \frac{\sin 1}{\cos 1} \cdots \frac{\sin 89}{\cos 89}$

$\frac{\cos 1}{\sin 1}$

② $\log_{10} \tan 1 \tan 2 \cdots \tan 88 \tan 89$

$\cot 2 \quad \cot 1$

$\tan 45$

$\log_{10} 1 = 0$

Exercise 10 Let $a \geq b > 1$. What is the largest possible value of

$$\log_a \frac{a}{b} + \log_b \frac{b}{a} ?$$

(A) -2 **(B)** 0 **(C)** 2 **(D)** 3 **(E)** 4

$(\log_a a - \log_a b) + (\log_b b - \log_b a)$

$(1 - \log_a b) + (1 - \log_b a)$

$2 - \log_a b - \log_b a$

$x > 0$

$2 - x - \frac{1}{x}$

$0 < \log_a b = \frac{1}{\log_b a}$

$\frac{\log b}{\log a} = \frac{1}{\frac{\log a}{\log b}}$

$= \frac{2x - x^2 - 1}{x}$

$= \frac{-x^2 + 2x - 1}{x}$

$= -\frac{(x-1)^2}{x} \leq 0$

$\log_a 1 = 0$

$\log_a b > 0$

if $b > 1$

$x \overset{?}{=} 1$

$\log_a b \overset{?}{=} 1$ if $a = b$