Exercise 4 Let $f(n) = \log_{2002} n^2$ for all positive integers n. Define

$$N = f(11) + f(13) + f(14).$$

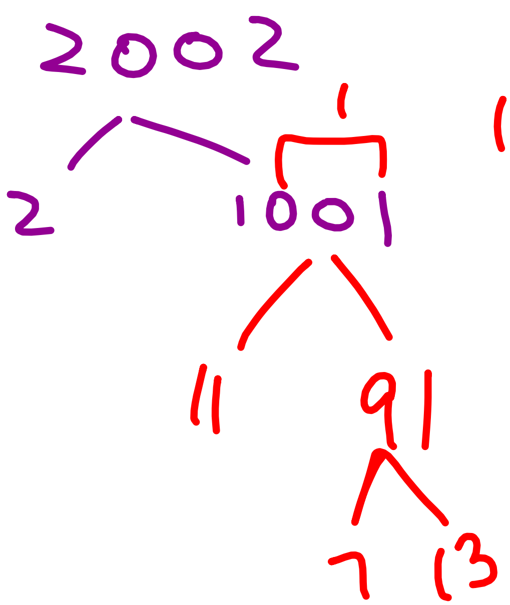
$2 \log_{2002} 11 \cdot 13 \cdot 14$

$2 \cdot 1$

2

Which of the following relations is true?

(A) $N > 1$ **(B)** $N = 1$ **(C)** $1 < N < 2$ **(D)** $N = 2$
(E) $N > 2$



$1 - 1 = 0 \rightarrow$ div. by 11

$\frac{1001}{11}$

Exercise 5 For some real numbers a and b, the equation

$$8x^3 + 4ax^2 + 2bx + a = 0$$

has three distinct positive roots, and the sum of the base-2 logarithms of the roots is 5. What is the value of a?

(A) -256 **(B)** -64 **(C)** -8 **(D)** 64 **(E)** 256

$\log_2 r_1 + \log_2 r_2 + \log_2 r_3 = 5$

$r_1 \cdot r_2 \cdot r_3 = 2^5$

$= 32$

$\frac{-a}{8} = 32$

$a = -256$

$$ax^3+bx^2+cx+d=0$$

$$r_1+r_2+r_3=-\frac{b}{a}$$

$$r_1r_2+r_1r_3+r_2r_3=\frac{c}{a}$$

$$r_1r_2r_3=-\frac{d}{a}$$

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Exercise 6 For any positive integer n, define

$$f(n) = \begin{cases} \log_8 n, & \text{if } \log_8 n \text{ is rational,} \\ 0, & \text{otherwise.} \end{cases}$$

$8^{\frac{a}{b}} = n$:

$2^{\frac{3a}{b}} = n$

What is $\sum_{n=1}^{1997} f(n)$?

(A) $\log_8 2047$ **(B)** 6 **(C)** $\frac{55}{3}$ **(D)** $\frac{58}{3}$ **(E)** 585

$\frac{1}{3} + \frac{2}{3} + \cdots + \frac{10}{3} = \frac{55}{3}$

Exercise 7 What is the value of the expression

$$N = \frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \cdots + \frac{1}{\log_{100} 100!}?$$

(A) 0.01 **(B)** 0.1 **(C)** 1 **(D)** 2 **(E)** 10

$$\frac{1}{\frac{\log 100!}{\log 2}} \cdots = \frac{\log 2}{\log 100!} + \frac{\log 3}{\log 100!} \cdots$$

$$\frac{\log 100!}{\log(100!)} = 1 \qquad \frac{\log(2 \cdot 3 \cdot 4 \ldots 100)}{\log(100!)}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \binom{n+1}{2}$$