

logarithm \rightarrow exponents.

$$\log xy = \log x + \log y$$

$$\log \frac{x}{y} = \log x - \log y$$

$$\log x^y = y \cdot \log x$$

$$a^b \cdot a^c = a^{b+c}$$

$$\frac{a^b}{a^c} = a^{b-c}$$

$$(a^b)^c = a^{bc}$$

$$b = \log_a c$$

$$a^b = c$$

MISC. PROP. of log.

$$\log_{10} X = \log X$$

$$\log X, x > 0$$

if $\log X \in \mathbb{R}$

$$\log_e X = \ln X$$

$$\log_a a = 1$$

$$\log_a 0 \rightarrow \text{und.}$$

$$\log_a 1 = 0$$

$$\frac{\log_3 9}{\log_4 2} = 4$$

$$\log_2 9 \cdot \log_3 4 = 4$$

Prove

$$\log_a b = \frac{\ln b}{\ln a}$$

$$\log_a b = \frac{\log_e b}{\log_e a}$$

$$\log_a b \cdot \log_e a = \log_e b$$

$$\cancel{\log_a a} \cdot \log_e b = \log_e b$$

$$\log_a b = k \quad \Leftrightarrow \quad \frac{\ln b}{\ln a}$$
$$a^k = b$$
$$\ln a^k = \ln b$$
$$k \ln a = \ln b$$
$$k = \frac{\ln b}{\ln a}$$

The diagram illustrates the derivation of the change of base formula for logarithms. It starts with the equation $\log_a b = k$ and shows its equivalence to $\frac{\ln b}{\ln a}$. Below this, it shows the steps: $a^k = b$, $\ln a^k = \ln b$, $k \ln a = \ln b$, and finally $k = \frac{\ln b}{\ln a}$. Arrows indicate the logical flow and equivalence between these steps.

$$\log_a b \cdot \log_c d = \log_c b \cdot \log_a d$$
$$\frac{\log b}{\log a} \cdot \frac{\log d}{\log c} = \frac{\log b}{\log c} \cdot \frac{\log d}{\log a}$$

$$\log_2 \left(\log_4 \left(\log_8 (x) \right) \right) = 1$$

Find x .

$$\log_4 \left(\log_8 (x) \right) = 2$$

$$\log_8 (x) = 16$$

$$\textcircled{2^{48}}$$