

Exercise 9 For how many values of the coefficient a do the equations

$$0 = x^2 + ax + 1 \quad \text{and} \quad 0 = x^2 - x - a$$

have a common real solution?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

$$\begin{array}{l}
 ax + 1 = -x - a \\
 ax + a + x + 1 = 0 \\
 (a+1)(x+1) = 0 \\
 \quad \quad \quad \underline{x = -1} \\
 0 = 1 - a + 1 \\
 a = 2 \leftarrow
 \end{array}
 \quad \left| \quad \begin{array}{l}
 a = -1 \\
 0 = x^2 - x + 1 \\
 x \text{ is not real} \\
 0 = 1 + 1 - a
 \end{array}
 \right.$$

$$h(x) = f(g(x))$$

$$\textcircled{1} \quad f(x) = ?$$

$$h(g^{-1}(x)) = f(g(g^{-1}(x)))$$

$$h(g^{-1}(x)) = f(x)$$

$$\textcircled{2} \quad g(x) = ?$$

$$f^{-1}(h(x)) = f^{-1}(f(g(x)))$$

$$f^{-1}(h(x)) = g(x)$$

$$h(x) = f(g(x))$$

$$\textcircled{1} \quad h(x) = 2x - 5$$

$$g(x) = \sqrt{x+5} - 4 \quad \xrightarrow{g^{-1}(x)} \quad = (x+4)^2 - 5$$

$$\begin{aligned} \text{Find } f(x) &= h(g^{-1}(x)) \\ &= 2((x+4)^2 - 5) - 5 \end{aligned}$$

$$\textcircled{2} \quad h(x) = 3\sqrt{x-5} \quad \xrightarrow{f^{-1}(x)}$$

$$f(x) = \sin(x+5) \quad \xrightarrow{f^{-1}(x)} \quad = (\arcsin x) - 5$$

$$g(x) = f^{-1}(h(x)) = \arcsin(3\sqrt{x-5}) - 5$$

$$\arcsin\left(\overset{\ominus}{\sin \frac{4\pi}{3}}\right) \quad \arcsin\left(\sin \frac{\pi}{4}\right)$$

$$= -\frac{\pi}{3} \quad = \frac{\pi}{4}$$

$$-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}$$

$$\arcsin\left(\sin \frac{4\pi}{3}\right)$$

$$= \arcsin\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \left(-\frac{\pi}{3}\right)$$

$$-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}$$

$$0 \leq \arccos x \leq \pi$$

$$-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$$

$$\frac{5\pi}{3}$$

$$-\frac{\pi}{3}$$

$$\arctan\left(\tan \frac{7\pi}{6}\right) = \frac{7\pi}{6}$$

Find all roots for

$$3x^4 - x^3 - 9x^2 + 9x - 2 = 0.$$

$$\frac{\pm 1, \pm 2}{\pm 1, \pm 3} \rightarrow \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$$

$$\begin{array}{r} \downarrow \\ 3 \quad -1 \quad -9 \quad 9 \quad -2 \\ \quad 3 \quad 2 \quad -7 \quad 2 \\ \hline \downarrow \quad 3 \quad 2 \quad -7 \quad 2 \quad | \quad 0 \\ \quad 3 \quad 5 \quad -2 \\ \hline 3 \quad 5 \quad -2 \quad | \quad 0 \end{array}$$

$$3x^2 + 5x - 2$$

$$= (3x - 1)(x + 2)$$

$$\frac{1}{3}, -2$$

$$x = \left\{ 1, -2, \frac{1}{3} \right\}, \quad 1 \text{ is mult. of } (2)$$