Exercise 4 What is the maximum number of points of intersection of the graphs of two different fourth-degree polynomial functions $y = P(x)$ and $y = Q(x)$, each with leading coefficient 1?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

$P(x) = x^4 + ax^3 + bx^2 + cx + d$

$Q(x) = x^4 + ex^3 + fx^2 + gx + h$

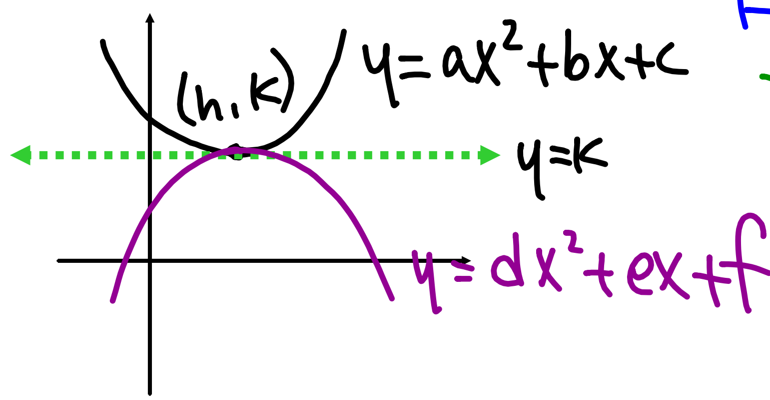
$\cancel{x^4} + ax^3 + bx^2 \ldots = \cancel{x^4} + ex^3 \ldots$

$ax^3 - ex^3 + bx^2 - fx^2 \ldots = 0$

$(a-e)x^3 \ldots x^2$

Exercise 5 The parabola with equation $y(x) = ax^2 + bx + c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $y_r(x) = dx^2 + ex + f$. Which of the following equals $\cancel{a} + \cancel{b} + c + \cancel{d} + \cancel{e} + f$?

(A) $2b$ **(B)** $2c$ **(C)** $2a + 2b$ **(D)** $2h$ $\boxed{\textbf{(E)}\ 2k}$



$f + c = 2k$

$a = -d$

$b = e$

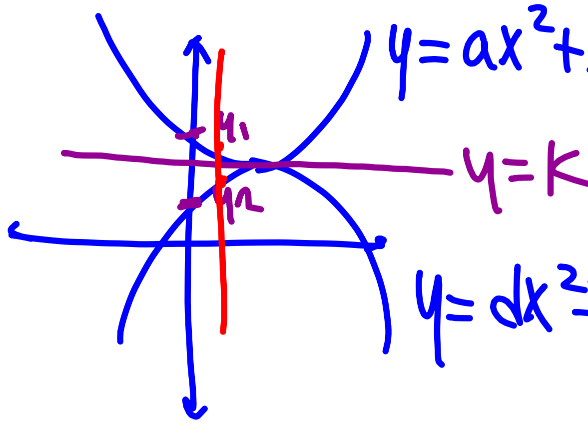
$\frac{-b}{2a} = \frac{-e}{2d}$

$\frac{-b}{2a} = \frac{+e}{2(-a)}$

Exercise 5 The parabola with equation $y(x) = ax^2 + bx + c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $y_r(x) = dx^2 + ex + f$. Which of the following equals $a + b + c + d + e + f$?

(A) $2b$ **(B)** $2c$ **(C)** $2a + 2b$ **(D)** $2h$ **(E)** $2k$

$y(1)$



$y = ax^2 + bx + c \Big|_{x=1} = a + b + c$

$\frac{y_1 + y_2}{2} = K$

$y_1 + y_2 = 2k$

$y = dx^2 + ex + f \Big|_{x=1} = d + e + f$

$y(1)$

Exercise 6 Let $P(x)$ be a polynomial which when divided by $x - 19$ has the remainder 99, and when divided by $x - 99$ has the remainder 19. What is the remainder when $P(x)$ is divided by $(x - 19)(x - 99)$?

(A) $-x + 80$ **(B)** $x + 80$ **(C)** $-x + 118$ **(D)** $x + 118$ **(E)** 0

$P(19) = 99$

$P(99) = 19$

$P(x) = D(x)Q(x) + R(x)$

$= (x-19)(x-99)\,Q(x) + mx + b$

$P(19) = 0 + 19m + b = 99$

$P(99) = 99m + b = 19$

$-80m = 80$

$b = 118 \quad m = -1$

Exercise 7 The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the average of its zeros, the product of its zeros, and the sum of its coefficients are all equal. The y-intercept of the graph of $y = P(x)$ is 2. What is b?

(A) -11 (B) -10 (C) -9 (D) 1 (E) 5

$c=2$

$a=6$

$$-\frac{a}{3} = \overset{-2}{\cancel{-\frac{c}{1}}} = 1+a+b+c$$

$-2=1+6+b+2$

$-11=b$

$$\overset{+}{a}x^3 \overset{-}{+} bx^2 \overset{+}{+} cx \overset{-}{+} d = 0$$

$$P = \frac{-d}{a}$$