Exercise 1 Let $P(x)$ be a linear polynomial with $P(6) - P(2) = 12$. What is $P(12) - P(2)$?

(A) 12 (B) 18 (C) 24 (D) 30 (E) 36

$$\frac{\Delta y}{\Delta x} = \frac{\boxed{30}}{10}$$

$\Delta y = 12$

$\Delta x = 4$

Exercise 2 Let $x_1 \neq x_2$ be such that $3x_1^2 - hx_1 = b$ and $3x_2^2 - hx_2 = b$. What is $x_1 + x_2$?

(A) $-\frac{h}{3}$ (B) $\frac{h}{3}$ (C) $\frac{b}{3}$ (D) $2b$ (E) $-\frac{b}{3}$

$3x_1^2 - hx_1 - b = 0$

$3x_2^2 - hx_2 - b = 0$

x_1 & x_2 are roots of

$y = 3x^2 - hx - b$

$S = \frac{-b}{a} = \frac{-(-h)}{3} = \frac{h}{3}$

Exercise 2 Let $x_1 \neq x_2$ be such that $3x_1^2 - hx_1 = b$ and $3x_2^2 - hx_2 = b$. What is $x_1 + x_2$?

(A) $-\frac{h}{3}$ **(B)** $\frac{h}{3}$ **(C)** $\frac{b}{3}$ **(D)** $2b$ **(E)** $-\frac{b}{3}$

$$3x_1^2 - 3x_2^2 = hx_1 - hx_2$$

$$3(x_1^2 - x_2^2) = h(x_1 - x_2)$$

$$3(x_1 + x_2)(x_1 - x_2) = \text{''}$$

$$3(x_1 + x_2) = h$$

$$x_1 + x_2 = \frac{h}{3}$$

Exercise 3 What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

(A) 0 **(B)** 1 **(C)** 49 **(D)** 50 **(E)** 51

$$(-1)^{51} + 51$$

$$51 - 1 = 50$$

$$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 0 & 0 & \cdots & 51 \\ & & -1 & 1 & -1 & 1 & \\ \hline & 1 & -1 & 1 & -1 & & \end{array}$$

27. $P(x) = x^5 - 6x^3 - x^2 + 2x + 18$

Poss. roots $= \pm 1, \pm 18, \pm 2, \pm 9, \pm 3, \pm 6,$

$$\begin{array}{r|rrrrrr} -3 & 1 & 0 & -6 & -1 & 2 & 18 \\ & & -9 & 27 & & & \\ \hline & 3 & -9 & 21 & & & \end{array}$$

49. Does there exist a polynomial of degree 4 with integer coefficients that has zeros i, $2i$, $3i$, and $4i$? If so, find it. If not, explain why.

① $(x-i)(x-2i)(x-3i)(x-4i)$ & multiply

$ax^4 + bx^3 + cx^2 + dx + e$

$r_1 + r_2 + r_3 + r_4 = -\frac{b}{a}$

$r_1r_2 + r_1r_3 + \cdots + r_3r_4 = \frac{c}{a}$

sum of roots

$10i \neq -\frac{b}{a}$, where $a, b \in \mathbb{Z}$

So, there is no such polynomial.