118. If $f\left(\frac{x}{x-1}\right) = \frac{1}{x}$ for all $x \neq 0, 1$ and $0 < \theta < \frac{\pi}{2}$, then find $f(\sec^2\theta)$. (AHSME 1991)

① $\frac{x}{x-1} = y \xrightarrow{\text{inv}} \frac{y}{y-1} = x$

$y = xy - x$

$y(1-x) = -x$

$y = \frac{-x}{1-x}$

$f(x) = \frac{x-1}{x}$

$f(x) = 1 - \frac{1}{x}$

$x = \sec^2\theta$

$= 1 - \cos^2\theta = \underline{\sin^2\theta}$

118. If $f\left(\frac{x}{x-1}\right) = \frac{1}{x}$ for all $x \neq 0, 1$ and $0 < \theta < \frac{\pi}{2}$, then find $f(\sec^2\theta)$. (AHSME 1991)

$\frac{1}{\csc^2\theta} = \boxed{\sin^2\theta}$

$\frac{x-1+1}{x-1} = \sec^2\theta$

$\frac{1}{x-1} = \sec^2\theta - 1$

$= \tan^2\theta$

$1 + \frac{1}{x-1} = \sec^2\theta$

$x-1 = \cot^2\theta$

$x = 1 + \cot^2\theta$

$= \csc^2\theta$

33. If $f(x) = \log_2 x$ for $x > 0$, then $f^{-1}(x) =$

(A) 2^x

(B) x^2

(C) $\frac{x}{2}$

(D) $\frac{2}{x}$

(E) $\log_x 2$

$y = \log_2 x \leftrightarrow x = 2^y$

inv

$y = 2^x$

$(2x^3 - 6x + 5) \div (x - 2)$

$= 2x^2 + 4x + 2 + \frac{9}{x-2}$

① Long Div.

$$\begin{array}{r|l} & 2x^2 + 4x + 2 \\ \hline x - 2 & 2x^3 + 0x^2 - 6x + 5 \\ & 2x^3 - 4x^2 \\ \hline & 4x^2 - 6x \\ & 4x^2 - 8x \\ \hline & 2x + 5 \\ & 2x - 4 \\ \hline & 9 \end{array}$$

2	2	0	-6	5
	↓	4	8	4
	2	4	2	9

$\frac{P(x)}{x-a}$, Rem: $P(a)$

Rem: $P(2) = 2(2)^3 - 6(2) + 5 = \underline{\underline{9}}$

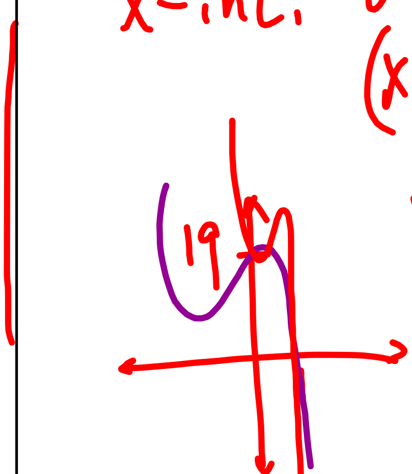
1. Graph the polynomial $P(x) = -(x+2)^3 + 27$, showing clearly all x- and y-intercepts.

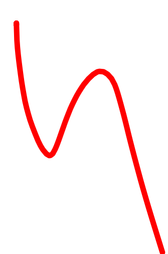
y-int: $P(0) = -(2)^3 + 27 = 19$ y-Int $\underline{\underline{19}}$

x-int: $0 = -(x+2)^3 + 27$ x-int

$(x+2)^3 = 27$ $x = 1$

$x + 2 = \sqrt[3]{27}$





(b) Use long division to find the quotient and remainder when $2x^5 + 4x^4 - x^3 - x^2 + 7$ is divided by $2x^2 - 1$.

$$2(x^2 - \tfrac{1}{2}) = 2\left(x - \tfrac{1}{\sqrt{2}}\right)\left(x + \tfrac{1}{\sqrt{2}}\right)$$

$$\begin{array}{c|cccccc}
\frac{1}{\sqrt{2}} & 2 & 4 & -1 & -1 & 0 & 7 \\
 & & \frac{2}{\sqrt{2}} & \frac{4}{\sqrt{2}}+1 & 2 & \frac{1}{\sqrt{2}} & \frac{1}{2} \\
\hline
 & 2 & 4+\frac{2}{\sqrt{2}} & \frac{4}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} & 7.5
\end{array}$$