Exercise 3 Let $f(x) = ax^7 + bx^3 + cx - \cancel{5}$, where a, b, and c are constants. Suppose that $f(-7) = 7$. What is $f(7)$?

(A) -17 **(B)** -7 **(C)** 14 **(D)** 17 **(E)** 21

x^0

$$g(x) = f(x) + 5$$

$$g(-7) = f(-7) + 5$$
$$= 7 + 5 = 12$$

$$g(7) = f(7) + 5$$
$$-12 = f(7) + 5$$
$$-17 = f(7)$$

since odd,

$$g(-x) = -g(x)$$
$$g(-7) = -g(7)$$
$$12 = -g(7)$$
$$g(7) = -12$$

Exercise 4 The function f satisfies $f(2+x) = f(2-x)$ for all real numbers x. Moreover, $f(x) = 0$ has exactly four distinct real roots. What is the sum of these roots?

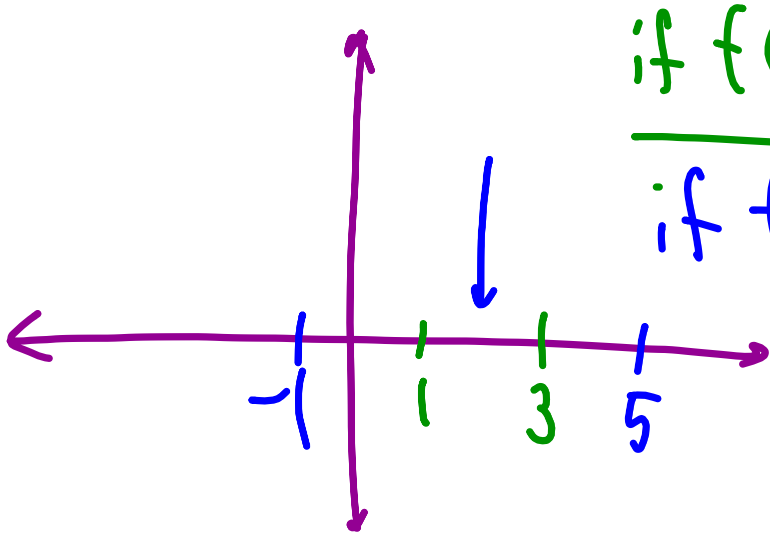
(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

$x = -1$

if $f(1) = 0 = f(3)$

if $f(5) = 0 = f(-1)$

$x = 3$



Exercise 5 Suppose that the function f, for $x \neq -3/2$, is defined by

$$f(x) = \frac{cx}{2x+3},$$

and that $f(f(x)) = x$ for all real numbers in its domain. What is the value of c?

(A) -3 **(B)** $-\frac{3}{2}$ **(C)** $\frac{3}{2}$ **(D)** 3 **(E)** 5

$$\frac{(2x+3)\left(c\left(\frac{cx}{2x+3}\right)\right)}{(2x+3)\left(2\left(\frac{cx}{2x+3}\right)+3\right)} = x \rightarrow \frac{c^2x}{2cx+6x+9} = x$$

$$0x^2 + c^2x = 2cx^2 + 6x^2 + 9x$$

$$\left.\begin{aligned} 0 = 2c+6 &\rightarrow c=-3 \\ c^2=9 &\rightarrow c=\pm 3 \end{aligned}\right\} \quad c=-3$$

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(A) -3 (B) $-\frac{3}{2}$ (C) $\frac{3}{2}$ (D) 3 (E) 5

$$y = \frac{cx}{2x+3} \xrightarrow{\text{Inv}} x = \frac{cy}{2y+3}$$

$$2xy + 3x = cy$$

$$2xy - cy = -3x$$

$$y = \frac{-3x}{2x-c}$$

Exercise 4 The function f satisfies $f(2+x) = f(2-x)$ for all real numbers x. Moreover, $f(x) = 0$ has exactly four distinct real roots. What is the sum of these roots?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

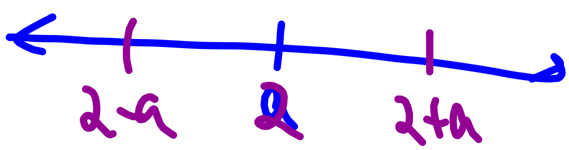
$x-2$

$$f(2+(x-2)) = f(2-(x-2))$$

sym. abt $x=2$ $\leftarrow$ $f(x) = f(4-x)$

$$f(x) = f(2a-x)$$

$$x = a$$



$f(g(x)) = x$

$\hookrightarrow$ f & g Inv.