10. (47%) If $a < b$, find the ordered pair of positive integers (a,b) that satisfies

$$\sqrt{10 + \sqrt{84}} = \sqrt{a} + \sqrt{b} \ .$$

$$10 + \sqrt{84} = a + 2\sqrt{ab} + b$$

$a+b=10$

$\sqrt{4ab}$

$4ab = 84$

$ab = 21$

$b=7,\ a=3$

$\boxed{(3,7)}$

16. (39%) If $a > 0$, $b > 0$, $a \neq b$, and $\dfrac{a\sqrt{b} + b\sqrt{a}}{a\sqrt{b} - b\sqrt{a}} - \dfrac{a\sqrt{b} - b\sqrt{a}}{a\sqrt{b} + b\sqrt{a}} = \sqrt{ab}$, write an equation expressing a explicitly in terms of b.

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$-\ (x-y)^2 = x^2 - 2xy + y^2$$

$$4xy$$

$$\frac{(a\sqrt{b} + b\sqrt{a})^2}{a^2b - b^2a} - \frac{(a\sqrt{b} - b\sqrt{a})^2}{a^2b - b^2a} = \sqrt{ab}$$

$$\frac{4\cancel{ab}\cancel{\sqrt{ab}}}{\cancel{ab}(a-b)} = \cancel{\sqrt{ab}} \rightarrow \frac{4}{a-b} = 1$$

$a - b = 4$

$a = 4 + b$

22. (32%) Find all ordered triples of real numbers (x, y, z) that satisfy

$$\sqrt{x-y+z} = \sqrt{x} - \sqrt{y} + \sqrt{z},$$
$$x + y + z = 8, \text{ and}$$
$$x - y + z = 4.$$

$2y = 4$

$y = 2$

$x + z = 6$

$2 = \sqrt{x} - \sqrt{2} + \sqrt{z}$

$2 + \sqrt{2} = \sqrt{x} + \sqrt{z}$

$6 + 4\sqrt{2} = x + z + 2\sqrt{xz}$

$2\sqrt{2} = \sqrt{xz}$

$8 = xz$

$(2, 2, 4)$

$(4, 2, 2)$