

7. Simplify

$$1 + \frac{3}{2 + \frac{3}{1 + \frac{3}{2 + \frac{3}{1 + \dots}}}} = X$$

$$1 + \frac{3}{2 + \frac{3}{X}} = X$$

$$X \neq 0, -\frac{3}{2}$$

$$\frac{X(3)}{X(2 + \frac{3}{X})} = X - 1 \rightarrow \frac{3X}{2X + 3} = X - 1$$

$$\rightarrow 3X = 2X^2 + X - 3$$

$$2X^2 - 2X - 3 = 0$$

$$X^2 - X + \frac{1}{4} = \frac{3}{2} + \frac{1}{4}$$

$$\left(X - \frac{1}{2}\right)^2 = \frac{7}{4}$$

$$X = \frac{1}{2} \pm \frac{\sqrt{7}}{2} \rightarrow \frac{1}{2} + \frac{\sqrt{7}}{2}$$

1. $\frac{[(n-1)!]^2}{[n!]^2}$

a. $\frac{1}{n}$

b. $\frac{1}{n^2}$

c. $\frac{n-1}{n}$

d. $\left(\frac{n-1}{n}\right)^2$

e. $(n-1)^2$

$$\frac{(n-1)! (n-1)!}{n! n!}$$

$$= \frac{\cancel{(n-1)!} \cancel{(n-1)!}}{n \cdot \cancel{(n-1)!} \cdot n \cdot \cancel{(n-1)!}}$$

$$= \frac{1}{n^2}$$

$$n! = \underbrace{n \cdot (n-1)(n-2)(n-3)(n-4) \dots}_{(n-1)!}$$

6. Simplify

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}$$

$$3 + \frac{1}{x} = x$$

$$x^2 - 3x - 1 = 0$$

$$x^2 - 3x + \frac{9}{4} = +1 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{13}{4}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

$$\left(\frac{3}{2} + \frac{\sqrt{13}}{2}\right)$$

2. Under which of the following condition

is $\frac{a-b}{ab}$ positive?

- a. $0 < a < b$
- b. $a < b < 0$
- c. $b < a < 0$
- d. $b < 0 < a$
- e. None of the above

If $ab < 0$

$$a < 0 < b$$

$$\frac{a-b}{ab} = \frac{1}{b} - \frac{1}{a} > 0$$

$$\frac{1}{b} > \frac{1}{a}$$

If $ab > 0$

$$\underline{a} > \underline{b} > 0$$

$$0 > \underline{a} > \underline{b}$$

6. For how many positive integers n is $n^2 - 3n + 2$ a prime number?

- (A) none (B) one (C) two (D) more than two, but finitely many
 (E) infinitely many

factor

$ab \rightarrow 1 \text{ or } 1$

20. Suppose that the number a satisfies the equation $4 = a + a^{-1}$. What is the value of $a^4 + a^{-4}$?

- (A) 164 (B) 172 (C) 192 (D) 194 (E) 212

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

1. The ratio $\frac{2^{2001} \cdot 3^{2003}}{6^{2002}}$ is

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{2}$

$$6 = 2 \cdot 3$$