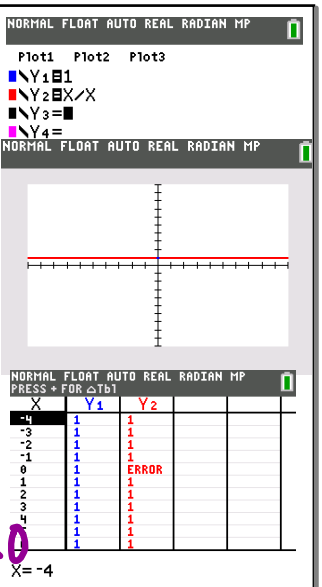
Rational expressions

1) Compare

$y = 1$ vs $y = \frac{x}{x}$

$y = 1$, except when $x = 0$



compare

$y = \frac{x^2 - 4}{x + 2}$ vs $x - 2 = y$

$y = x - 2, \; x \neq -2$

23. $\dfrac{x^2+7x+12}{x^2+3x+2} \cdot \dfrac{x^2+5x+6}{x^2+6x+9}$

$\dfrac{(x+3)(x+4)}{(x+2)(x+1)} \cdot \dfrac{(x+2)(x+3)}{(x+3)(x+3)}$ $\quad x \neq -3, -2$

$= \dfrac{x+4}{x+1}$, $x \neq -3, -2$

27. $\dfrac{\dfrac{x^3}{x+1}}{\dfrac{x}{x^2+2x+1}}$

$\dfrac{x^3}{x+1} \cdot \dfrac{(x+1)(x+1)}{x}$

$= x^2(x+1), x \neq 0, -1$

43. $\dfrac{2}{x+3} - \dfrac{1}{x^2+7x+12}$

$\dfrac{(x+4)2}{(x+4)(x+3)} - \dfrac{1}{(x+3)(x+4)}$ $\quad x \neq -4$

$\dfrac{2x+7}{(x+3)(x+4)}$, $x \neq -4$

57. $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}$

$\dfrac{\left(\frac{1}{x^2}-\frac{1}{y^2}\right)x^2y^2}{\left(\frac{1}{x}+\frac{1}{y}\right)x^2y^2} = \dfrac{y^2-x^2}{xy^2+x^2y}$ $\quad x \neq 0$, $y \neq 0$, $y \neq -x$

$= \dfrac{(y-x)(y+x)}{xy(y+x)} = \dfrac{y-x}{xy}$

$\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} \cdot \dfrac{(x^{-1}+y^{-1})(x^{-1}-y^{-1})}{(x^{-1}+y^{-1})} = x^{-1}-y^{-1}$

$\left(\frac{1}{x}-\frac{1}{y}\right) = \frac{y-x}{xy}$

67. $\dfrac{3(x+2)^2(x-3)^2-(x+2)^3(2)(x-3)}{(x-3)^4}$

$$\frac{(x+2)^2(x-3)\left(3(x-3)-(x+2)2\right)}{(x-3)^4}$$