110. The Power of Algebraic Formulas Use the Difference of Squares Formula to factor $17^2 - 16^2$. Notice that it is easy to calculate the factored form in your head, but not so easy to calculate the original form in this way. Evaluate each expression in your head:

(a) $528^2 - 527^2$ **(b)** $122^2 - 120^2$ **(c)** $1020^2 - 1010^2$

Now use the Special Product Formula

$$(A + B)(A - B) = A^2 - B^2$$

to evaluate these products in your head:

(d) $79 \cdot 51$ **(e)** $998 \cdot 1002$

$(1000-2)(1000+2)$

999996

$79 \quad 51$

$(65+14)(65-14)$

$65^2 - 14^2$

$= \frac{16900}{4} - 196$

$= 4225 - 196 = 4029$

89. $x^6 - 8y^3$

$= (x^2)^3 - (2y)^3$

$= (x^2 - 2y)(x^4 + 2x^2y + 4y^2)$

111. Differences of Even Powers

(a) Factor the expressions completely: $A^4 - B^4$ and $A^6 - B^6$.

(b) Verify that $18{,}335 = 12^4 - 7^4$ and that $2{,}868{,}335 = 12^6 - 7^6$.

(c) Use the results of parts (a) and (b) to factor the integers 18,335 and 2,868,335. Then show that in both of these factorizations, all the factors are prime numbers.

$$A^6 - B^6$$
$$=(A^3+B^3)(A^3-B^3)$$
$$=(A+B)(A^2-AB+B^2)(A-B)(A^2+AB+B^2)$$

$$A^6 - B^6 \quad (A^2+B^2)^2 - A^2B^2$$
$$=(A^2-B^2)(A^4+A^2B^2+B^4+A^2B^2-A^2B^2)$$
$$=(A+B)(A-B)(A^2+B^2+AB)(A^2+B^2-AB)$$

$$2868335 = 12^6 - 7^6$$
$$=(12+7)(12-7)(12^2+7^2+7\cdot 12)(12^2+7^2-7\cdot 12)$$
$$= 19\cdot 5\cdot 277\cdot 109$$

68. $x^{-3/2} + 2x^{-1/2} + x^{1/2}$

$*$ if CF $= x^{\frac{1}{2}}$

$$x^{\frac{1}{2}}\left(x^{-2} + 2x^{-1} + 1\right)$$

CF $\Rightarrow x^{-\frac{3}{2}}$

$$x^{-\frac{3}{2}}(1 + 2x + x^2) = \frac{(x+1)^2}{x^{\frac{3}{2}}} = \frac{(x+1)^2}{x^{1\frac{1}{2}}}$$

$$= \frac{(x+1)^2}{x\sqrt{x}}$$

70. $\underbrace{2x^{1/3}(x-2)^{2/3}} - \underbrace{5x^{4/3}(x-2)^{-1/3}}$

$$= x^{\frac{1}{3}}(x-2)^{-\frac{1}{3}}\left(2(x-2) - 5x\right)$$

$$\frac{\sqrt[3]{x}}{\sqrt[3]{x-2}}(-3x-4)$$

13

1 2 3 5 0 6 9

13