6. The product of four positive integers a, b, c, and d is 8!, and they satisfy the equations

$$ab + a + b = 524$$
$$bc + b + c = 146$$
$$cd + c + d = 104$$

What is $a - d$?

a) 4 b) 6 c) 8 d) 10 e) 12

5(105)

$(a+1)(b+1) = 525$

$(25)(21) = 3 \cdot 5^2 \cdot 7$

$(b+1)(c+1) = 147$ $7 \cdot 21$

$(21)(7) = 3 \cdot 7^2$

$(c+1)(d+1) = 105 = 3 \cdot 7 \cdot 5$

$(7)(15)$

$a = 24$
$b = 20$
$c = 6$
$d = 14$

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

7. What is the smallest possible sum of two positive integers whose product is 999,996?

$1000000 - 4$

$1000^2 - 2^2 = (1002)(998)$

sum = 2000

Prime factorization of 3599.

$3600 - 1$

$60^2 - 1^2 = (60+1)(60-1)$

$61 \cdot 59$ $\quad 61 \quad 59$

67. $x^{5/2} - x^{1/2}$

$x^{\frac{1}{2}}(x^2 - 1) = x^{\frac{1}{2}}(x+1)(x-1)$

69. $(x^2 + 1)^{1/2} + 2(x^2 + 1)^{-1/2}$

$$(x^2+1)^{-\frac{1}{2}}\left((x^2+1)+2\right)$$

$$= (x^2+1)^{-\frac{1}{2}}(x^2+3) = \frac{x^2+3}{\sqrt{x^2+1}}$$

68. $x^{-3/2} + 2x^{-1/2} + x^{1/2}$

$$x^{-3/2}\left(1+2x+x^2\right)$$