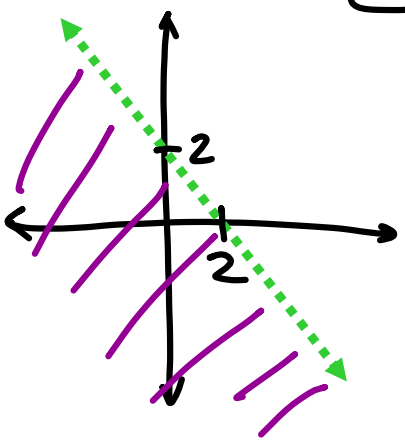
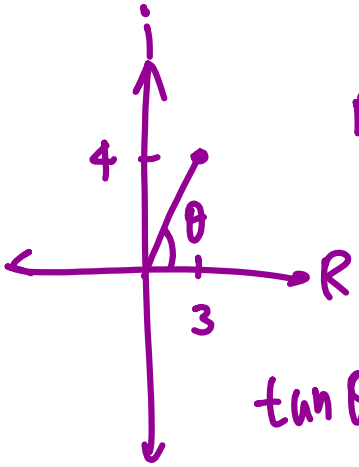
23. $\{z = a + bi \mid a + b < 2\}$



$x+y<2$

$y<-x+2$

39. $3 + 4i$



$r=5$ $\quad (5, \tan^{-1}\frac{4}{3})$

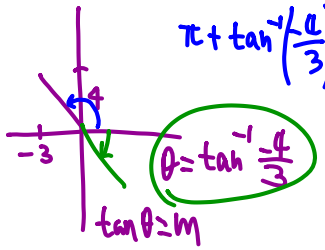
$\tan\theta = m \quad \theta = \tan^{-1}\frac{4}{3}$

$5 \operatorname{cis} \tan^{-1}\frac{4}{3}$

$-3+4i \xrightarrow{\text{Polar}} 5\,\text{cis}\left(\pi+\tan^{-1}\left(\frac{-4}{3}\right)\right)$

$0 \le \theta < 2\pi$

$0 \le r$

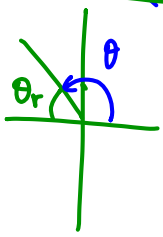


$\pi+\tan^{-1}\left(\frac{4}{3}\right)$

$5\cos\left(\pi+\tan^{-1}\left(\frac{-4}{3}\right)\right)+5i\sin\left(\pi+\tan^{-1}\left(\frac{-4}{3}\right)\right)$

ref θ

$\theta_r=\tan^{-1}\left(\frac{4}{3}\right)$

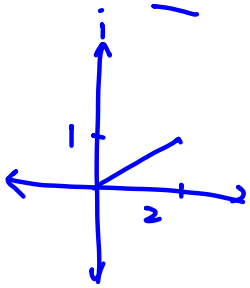


$\theta=\pi-\theta_r$

$=\pi-\tan^{-1}\frac{4}{3}$

$5\,\text{cis}\left(\pi-\tan^{-1}\frac{4}{3}\right)$

45. $2+i$



$r=\sqrt{5}$

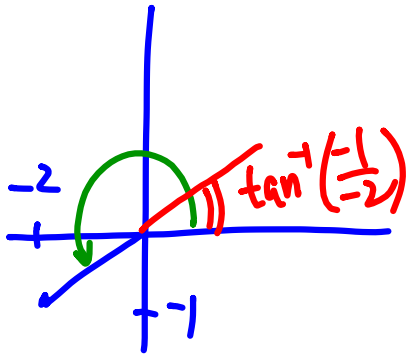
$\theta=\tan^{-1}\left(\frac{1}{2}\right)$

$\sqrt{5}\,\text{cis}\tan^{-1}\left(\frac{1}{2}\right)$

$$-2-i = \sqrt{5}\ \text{cis}\left(\pi + \tan^{-1}\left(\frac{1}{2}\right)\right)$$

$r = \sqrt{5}$

$\theta = \tan^{-1}\left(\frac{-1}{-2}\right) = \tan^{-1}\left(\frac{1}{2}\right)$



rec. Polar

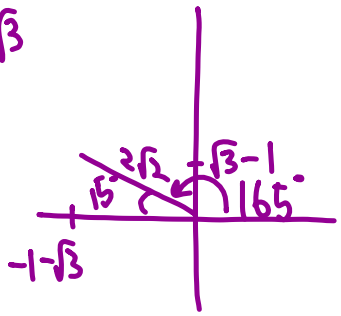
$z_1 = \sqrt{3} + i$ $\quad 2\ \text{cis}\ 30^\circ$

$z_2 = -1 + i$ $\quad \sqrt{2}\ \text{cis}\ 135^\circ$

$z_1 \cdot z_2 = -1-\sqrt{3}+(\sqrt{3}-1)i$ $\quad 2\sqrt{2}\ \text{cis}\ 165^\circ$

$r = \sqrt{4+2\sqrt{3}+4-2\sqrt{3}}$

$= 2\sqrt{2}$



$\sin\theta = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$$(r_1 \operatorname{cis}\theta_1)(r_2 \operatorname{cis}\theta_2)$$

Prove

$$= (r_1 r_2) \operatorname{cis}(\theta_1 + \theta_2)$$