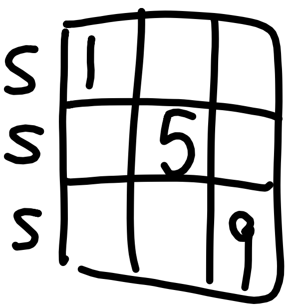
Problem 5

The 25 integers from -10 to 14, inclusive, can be arranged to form a 5-by-5 square in which the sum of the numbers in each row, the sum of the numbers in each column, and the sum of the numbers along each of the main diagonals are all the same. What is the value of this common sum?

(A) 2 **(B)** 5 **(C)** 10 **(D)** 25 **(E)** 50

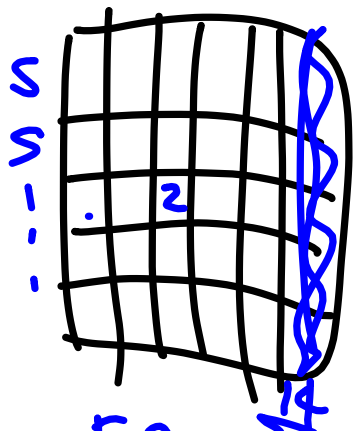


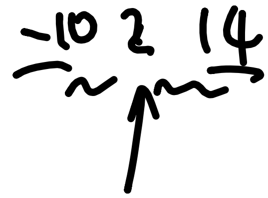
1 2 3
4 5 6
7 8 9
15

$3S = \sum 1 \to 9$

$3S = 45$

$S = 15$





$5S = \sum_{-10}^{14} i$

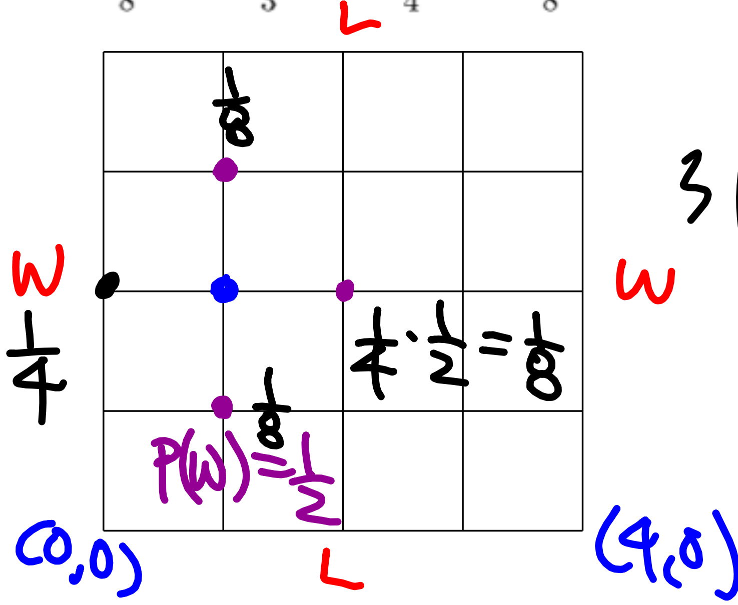
11
12
13
14
50

$S = 10$

Problem 11

A frog sitting at the point $(1, 2)$ begins a sequence of jumps, where each jump is parallel to one of the coordinate axes and has length 1, and the direction of each jump (up, down, right, or left) is chosen independently at random. The sequence ends when the frog reaches a side of the square with vertices $(0,0)$, $(0,4)$, $(4,4)$, and $(4,0)$. What is the probability that the sequence of jumps ends on a vertical side of the square?

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{5}{8} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{3}{4} \qquad \textbf{(E)}\ \frac{7}{8}$



$$3\left(\frac{1}{8}\right) + \frac{1}{4} = \frac{5}{8}$$

Problem 12

$m = \frac{3}{5}$

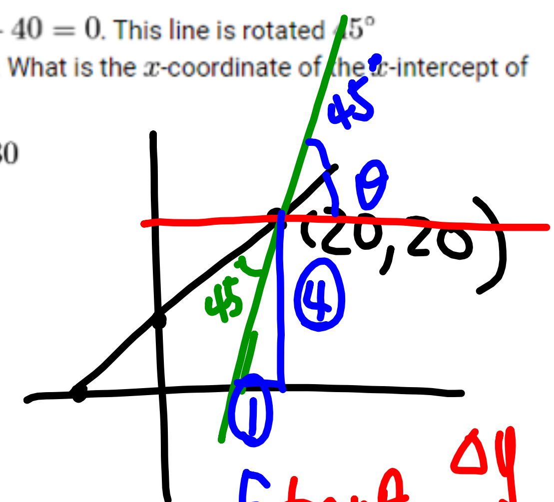
Line ℓ in the coordinate plane has the equation $3x - 5y + 40 = 0$. This line is rotated 45° counterclockwise about the point $(20, 20)$ to obtain line k. What is the x-coordinate of the x-intercept of line k?

(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

$$m_N = \tan(\theta + 45^\circ)$$

$$= \frac{\tan\theta + \tan 45}{1 - \tan\theta \tan 45}$$

$$= \frac{\frac{3}{5} + 1}{1 - \frac{3}{5}\cdot 1} = 4$$

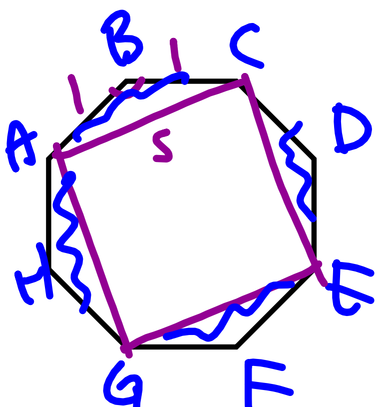


5 $\tan\theta = \frac{\Delta y}{\Delta x}$

Problem 14

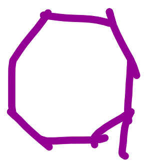
Regular octagon $ABCDEFGH$ has area n. Let m be the area of quadrilateral $ACEG$. What is $\frac{m}{n}$?

$\textbf{(A)}\ \frac{\sqrt{2}}{4} \qquad \textbf{(B)}\ \frac{\sqrt{2}}{2} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{3\sqrt{2}}{5} \qquad \textbf{(E)}\ \frac{2\sqrt{2}}{3}$



$$s^2 = 1^2 + 1^2 - 2\cdot 1\cdot 1 \cos 135^\circ$$

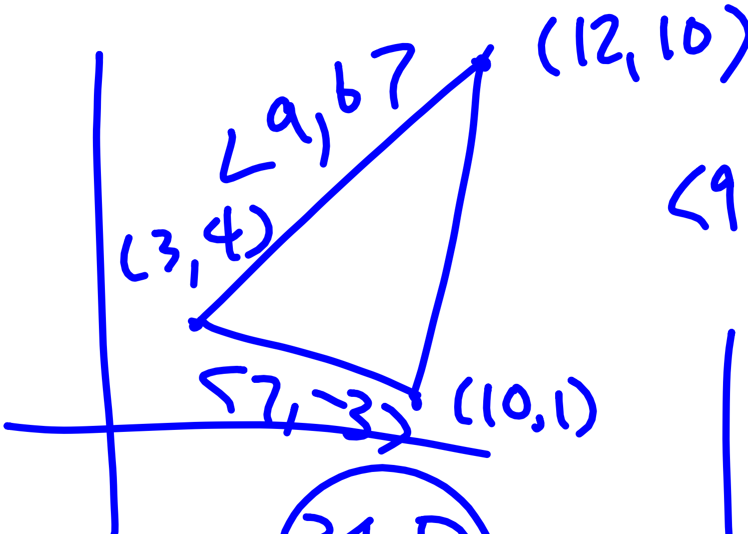
$$= 2+\sqrt{2}$$

 $= 2+\sqrt{2}+\sqrt{2} = 2+2\sqrt{2}$

$$\triangle = \frac{1}{2}\cdot 1\cdot 1 \sin 135^\circ = \frac{\sqrt{2}}{4}$$

$$\frac{(2+\sqrt{2})(1-\sqrt{2})}{(2+2\sqrt{2})(1-\sqrt{2})} = \frac{\cancel{2} - 2\sqrt{2} + \sqrt{2} - \cancel{2}}{2(-1)}$$

$$\frac{-\sqrt{2}}{-2} = \frac{\sqrt{2}}{2}$$



34.5

$\langle 9, 6, 0 \rangle \times \langle 7, -3, 0 \rangle$

$$\begin{vmatrix} 7 & -3 \\ 9 & 6 \end{vmatrix} = 42 + 27$$

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