E22

4. Prove

*you may work on only one side of the equation.

$$\frac{\tan a + \tan b}{\cot a + \cot b} = (\tan a)(\tan b)$$

$$\frac{(\tan a + \tan b)\tan a \tan b}{\left(\frac{1}{\tan a} + \frac{1}{\tan b}\right)\tan a \tan b}$$

$$\frac{(\tan a + \tan b)(\tan a \tan b)}{\tan b + \tan a} = \tan a \tan b$$

E21

5. Suppose that $P(2x) = x^2 + 3x - 1$. What is the product of all values of x for which $P\left(\frac{x}{4}\right) = 5$?

Sum $= \frac{-b}{a}$

$P\left(\frac{x}{4}\right) = \left(\frac{x}{8}\right)^2 + 3\left(\frac{x}{8}\right) - 1 = 5$

$\frac{x^2}{64} + \frac{3x}{8} - 6 = 0$

Prod $= \frac{-b}{\frac{1}{64}} = -384$

4. Let $P(x)$ be a polynomial which when divided by $x - 20$ has the remainder 19, and when divided by $x - 19$ has the remainder 20. What is the remainder when $P(x)$ is divided by $(x - 19)(x - 20)$?

$$Q(x) + \frac{ax+b}{(x-19)(x-20)}$$

by (20, 19)

(19, 20)

$$a = \frac{1}{-1} = -1$$

$y = -x + b$

$20 = -19 + b$

$39 = b$

$-x + 39$

$$y = \frac{3x^2+5x-8}{3x-1} = x+2 - \frac{6}{3x-1}$$

$y = x+2$

$\frac{1}{3}$ | 3 5 −8

1 2

3 6 −6

SA: $y = x+2$

HA: none

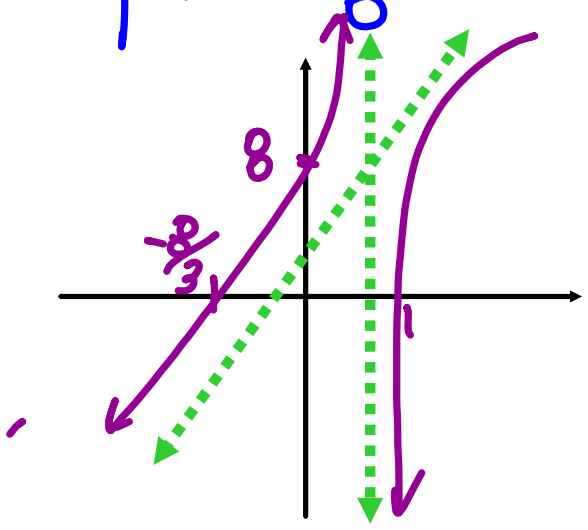
VA: $x = \frac{1}{3}$

x-int: $1, -\frac{8}{3}$

y-int: -8

$3x^2+5x-8=0$

$(x-1)(3x+8)$



Ex 11

7. Simplify, express without using factorial notation.

$$\frac{(n-2)!}{(n-4)!}$$

$$\frac{(n-2)(n-3)(n-4)!}{(n-4)!} = (n-2)(n-3)$$

Let $f(x) = \sqrt{4-x}$ and $g(x) = x^2 + 2x$.

5. Find domain and range of $f \circ g$.

$g(x) = x^2+2x$

$= (x+1)^2 - 1$

D: all real

R: $y \geq -1$

$f(x) = \sqrt{4-x} \geq 0$

D: $x \leq 4$

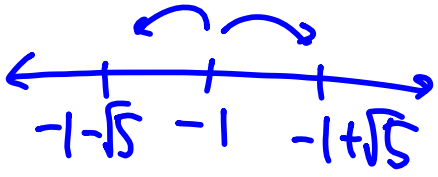
Domain of $f \circ g$

$-1 \leq (x+1)^2 - 1 \leq 4$

$0 \leq (x+1)^2 \leq 5$ → $|x+1| \leq \sqrt{5}$

$-\sqrt{5} \leq x+1 \leq \sqrt{5}$

$-\sqrt{5}-1 \leq x \leq \sqrt{5}-1$



Range $-\sqrt{5}-1 \leq x \leq \sqrt{5}-1$

$-1 \leq (x+1)^2 - 1 \leq 4$

$1 \geq 1-(x+1)^2 \geq -4$

$5 \geq 5-(\quad)^2 \geq 0$

$\sqrt{5} \geq \sqrt{\quad} \geq 0$