

Product-to-Sum Formulas

$$1) \sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$2) \cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$$3) \cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$$

$$4) \sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\begin{aligned} 54. \quad 3 \cos 37.5^\circ \cos 7.5^\circ &= \frac{3}{2} (\cos 45^\circ + \cos 30^\circ) \\ &= \frac{3}{2} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \right) = \frac{3(\sqrt{2} + \sqrt{3})}{4} \end{aligned}$$
$$\cos A \cos B = \frac{1}{2} (\cos (A+B) + \cos (A-B))$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\begin{aligned} & 2 \sin A \cos B \\ & 2 \left(\frac{1}{2} [\sin(A+B) + \sin(A-B)] \right) \\ & = \sin(A+B) + \sin(A-B) \\ & \sin x + \sin y \end{aligned}$$