31. $x^2 + y^2 + z^2 = 3xyz$

Find $\frac{\partial z}{\partial x}$

$$2x + 2z\frac{\partial z}{\partial x} = 3yz + 3yx\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{2x - 3yz}{3xy - 2z}$$

$$F = x^2 + y^2 + z^2 - 3xyz = 0 \qquad \frac{\partial z}{\partial x} \quad \frac{\partial F}{\partial z} \quad \frac{\partial F}{\partial x}$$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\boxed{\frac{\partial z}{\partial x}} = 0$$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}}$$

$$\frac{\partial z}{\partial x} = \frac{-(2x - 3yz)}{2z - 3xy}$$

33. $x - z = \arctan(yz)$

$$\frac{1}{1+\frac{y}{1+(yz)^2}}$$

$$\frac{\partial z}{dx} = \frac{-\frac{dF}{dx}}{\frac{dF}{dz}} = \frac{-1}{-1-\frac{y}{1+(yz)^2}} =$$

$f(x,y) = 2x - 3y \qquad u = \langle a, b \rangle$

$$D_u f = \lim_{h\to 0} \frac{f(x+ah,\, y+bh) - f(x,y)}{h}$$

$$\lim_{h\to 0} \frac{2(\cancel{x}+ah) - 3(\cancel{y}+bh) - (\cancel{2x} - \cancel{3y})}{h}$$

$$\lim_{h\to 0} \frac{2ah - 3bh}{h} = 2a - 3b$$

$$D_u f = \langle f_x, f_y \rangle \cdot u$$

$$= \nabla f \cdot u \qquad \nabla f = \langle f_x, f_y, \dots \rangle$$

$f(x,y) = x^2 + 2y^2$

Find $D_u f$ at $(2,1)$, $u = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$

$\nabla f = \langle 2x, 4y \rangle \Big|_{(2,1)}$

$= \langle 4, 4 \rangle$

$D_u f = \langle 4,4 \rangle \cdot \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$

$= \frac{12}{\sqrt{5}}$