21. $f(x, y) = \dfrac{x - y}{x + y}$

$\dfrac{(x+y)-(x-y)}{(x+y)^2}$

$\dfrac{2y}{(x+y)^2}$

Find f_x & f_y

$$\frac{\partial}{\partial y}\left(\frac{x-y}{x+y}\right) = \frac{-1(x+y)-1(x-y)}{(x+y)^2}$$

$$= \frac{-x-y-x+y}{(x+y)^2}$$

$$= \frac{-2x}{(x+y)^2}$$

46. $yz = \ln(x + z)$

$z = f(x,y)$

Find $\dfrac{\partial z}{\partial y}$

$$z + y\frac{\partial z}{\partial y} = \frac{1}{x+z}\frac{\partial z}{\partial y}$$

$$\frac{z}{\frac{1}{x+z} - y} = \frac{\partial z}{\partial y}$$

47. $x - z = \arctan(yz)$

Find $\frac{\partial z}{\partial x}$

$$1 - \frac{\partial z}{\partial x} = \frac{1}{1+(yz)^2} \cdot y \frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{1}{\frac{y}{1+(yz)^2} + 1}$$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx}$$

$$\frac{\partial^2 f}{\partial x \partial y} = f_{yx}$$

Tangent plane

$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$

$z-z_0 = a(x-x_0)+b(y-y_0)$ $\quad b \to \frac{\partial z}{\partial y}$

if $y=y_0 \to z-z_0=a(x-x_0)$ $\quad a \to \frac{\partial z}{\partial x}$

eq. of T plane

$z-z_0=f_x(x-x_0)+f_y(y-y_0)$

Let $z = 2x^2+3y^2-xy$

$2+3-1=4$

Find eq. of T. plane at $(1,1)$ $\quad (1,1,4)$

$\frac{\partial z}{\partial x} = 4x-y = 3$

$\frac{\partial z}{\partial y} = 6y-x = 5$

$z-4=3(x-1)+5(y-1)$

18. $\sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$

17–18 Verify the linear approximation at $(0, 0)$.

18. $\sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$

$z =$