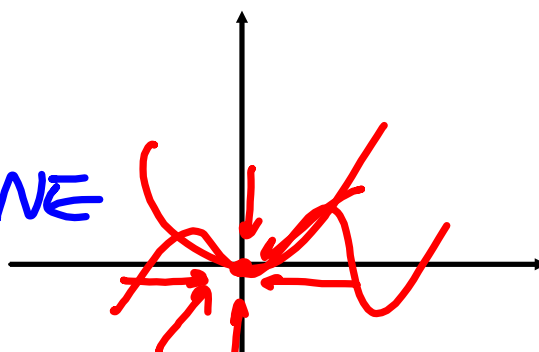


Evaluate

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2} = \text{DNE}$$



① x-axis: $y=0$

$$\lim_{(x,0) \rightarrow (0,0)} \frac{x^2}{x^2} = 1$$

② y-axis: $x=0$

$$\lim_{(0,y) \rightarrow (0,0)} \frac{-y^2}{y^2} = -1$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$$

$$(x, -x) \rightarrow -\frac{1}{2}$$

$$(x, x) \rightarrow \frac{1}{2}$$

$$y = mx$$
$$\lim_{\substack{(x,mx) \\ (0,0)}} \frac{mx^2}{x^2 + m^2x^2} = \frac{m}{1+m^2}$$

$$\lim_{\rightarrow(0,0)} \frac{xy^2}{x^2+y^4}$$

$$x = \frac{y^2}{y^4}$$

$$\lim \frac{\frac{y^2}{y^4}}{\frac{y^2}{y^4} + y^4} = \frac{y^2}{2y^4} = \frac{1}{2}$$

$$y = mx$$

$$\lim \frac{m^2 x^3}{x^2 + m^4 x^4}$$

$$\frac{m^2 x}{1 + m^4 x^2} = 0$$

∴ DNE

$$\lim_{(x,y) \rightarrow (a,b)} \frac{3x^2y}{x^2+y^2}$$

$$\lim_{(x,y) \rightarrow (a,b)} f(x,y) = L$$

$$\text{if } 0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta, \quad |f(x,y) - L| < \epsilon$$

