$$\lim_{x \to 5} 2x - 3 = 7$$

cont.

$$\lim_{x \to c} f(x) = f(c)$$

?

Let $f(x)$ be a function defined on an interval that contains $x = a$, except possibly at $x = a$. Then we say that,

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is some number $\delta > 0$ such that

$$|f(x) - L| < \varepsilon \qquad \text{whenever} \qquad 0 < |x - a| < \delta$$

If $0 < |x - a| < \delta$, then $|f(x) - 4| < \varepsilon$

$\lim_{x \to 5} 2x-3=7$

Prove

Let $\delta=\frac{\varepsilon}{2}$

we know that

$|x-5|<\delta$

$2|x-5|<2\delta$

$|2x-10|<2\left(\frac{\varepsilon}{2}\right)$

$|(2x-3)-7|<\varepsilon$

$\therefore \lim_{x \to 5} 2x-3=7.$

SW

$|(2x-3)-7|<\varepsilon$

$|2x-10|<\varepsilon$

$2|x-5|<\varepsilon$

$|x-5|<\frac{\varepsilon}{2}$

$|x-5|<\delta$

$\lim_{x \to 5} 2x-3=6$

if $|x-5|<\delta$, $|f(x)-6|<\varepsilon$

$|2x-3-6|$

Let $\delta=\frac{\varepsilon}{2}$

we know that

$2|x-5|<(\delta)2$

$|2x-10|<2\delta=\varepsilon$

SW

$|2x-3-6|<\varepsilon$

$|2x-9|<\varepsilon$

$2|x-4.5|<\varepsilon$

$|x-5|<\delta$

$$\lim_{x\to 3} 5x-1 = 14$$

Prove ↗

Let $\delta = \frac{\varepsilon}{5}$

we know that

$$|x-3| < \delta$$

$$|5x-1-14| = |5x-15| = 5|x-3| < 5\delta = \varepsilon$$

$$|(5x-1)-14| < \varepsilon$$

$$\therefore \lim_{x\to 3} 5x-1 = 14$$

$$\lim_{x\to 1} x^2+3x = 4$$

$$|x^2+3x-4| < \varepsilon$$

$$|(x+4)(x-1)| < \varepsilon$$

$$\boxed{|x+4|}\,|x-1| < \varepsilon$$

$$|x-1| < \delta$$