21. A force with magnitude 20 N acts directly upward from the xy-plane on an object with mass 4 kg. The object starts at the origin with initial velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$. Find its position function and its speed at time t.

$F = \langle 0, 0, 20 \rangle \quad m = 4\,kg$

$a = \langle 0, 0, 5 \rangle \quad v_0 = \langle 1, -1, 0 \rangle$

$v = \int a\,dt$

$= \langle v_{0x}, v_{0y}, 5t + v_{0z} \rangle = \langle 1, -1, 5t \rangle$

$r = \int v\,dt = \langle t + r_{0x}, -t + r_{0y}, \frac{5}{2}t^2 + r_{0z} \rangle$

$r(t) = \langle t, -t, \frac{5}{2}t^2 \rangle$

$s(t) = |v(t)| = |\langle 1, -1, 5t \rangle|$

$= \sqrt{25t^2 + 2}$

23. A projectile is fired with an initial speed of 500 m/s and angle of elevation 30°. Find (a) the range of the projectile, (b) the maximum height reached, and (c) the speed at impact.

$V_0 = \langle 250\sqrt{3}, 250 \rangle$

$a = \langle 0, -10 \rangle$

$V = \int a\,dt = \langle 250\sqrt{3}, -10t + 250 \rangle$

$r = \langle 250\sqrt{3}t, -5t^2 + 250t \rangle$

$r = \langle R(t_0), 0 \rangle, \quad -5t^2 + 250t = 0$

range

$R(50) = 250\sqrt{3}(50)$

$= 12500\sqrt{3}$ m

$-5t(t - 50) = 0$

$t = 0, 50$

Max ht

$V(t) = \langle 250\sqrt{3}, -10t + 250 \rangle$

max ht, $V = \langle \_\_\_, 0 \rangle$

$-10t + 250 = 0$

$t = 25$

$r_y = -5(25)^2 + 250(25) = 3125$ m

$s(50) = |V(50)| = |\langle 250\sqrt{3}, -10(50) + 250 \rangle|$

$= 500$

29. A medieval city has the shape of a square and is protected by walls with length 500 m and height 15 m. You are the commander of an attacking army and the closest you can get to the wall is 100 m. Your plan is to set fire to the city by catapulting heated rocks over the wall (with an initial speed of 80 m/s). At what range of angles should you tell your men to set the catapult? (Assume the path of the rocks is perpendicular to the wall.)

$a = \langle 0, -10 \rangle$

$V_0 = \langle 80\cos\theta, 80\sin\theta \rangle$

$V = \langle 80\cos\theta, 80\sin\theta - 10t \rangle$

$r = \langle 80\cos\theta t, 80\sin\theta t - 5t^2 \rangle$

$r = \langle 100, 15 \rangle$

$80\cos\theta t = 100$

$t = \frac{5}{4\cos\theta}$

$15 = 80\sin\theta\left(\frac{5}{4\cos\theta}\right) - 5\left(\frac{5}{4\cos\theta}\right)^2$

$\theta = 13.1^\circ, 85.5^\circ$

$r = \langle 600, 15 \rangle$

$t = \frac{15}{2\cos\theta}$

$15 = 80\sin\left(\frac{15}{2\cos\theta}\right) - 5\left(\frac{15}{2\cos\theta}\right)^2$

$\theta = 37.8^\circ, 53.6^\circ$

$(13.1^\circ, 37.8^\circ) \cup (53.6^\circ, 85.5^\circ)$

41. The position function of a spaceship is

$$\mathbf{r}(t) = (3 + t)\,\mathbf{i} + (2 + \ln t)\,\mathbf{j} + \left(7 - \frac{4}{t^2 + 1}\right)\mathbf{k}$$

and the coordinates of a space station are (6, 4, 9). The captain wants the spaceship to coast into the space station. When should the engines be turned off?

$a = 0$

$r' = \langle 1, \frac{1}{t}, \frac{8t}{(t^2+1)^2} \rangle$

$(6,4,9) = (3+t, 2+\ln t, 7 - \frac{4}{t^2+1}) + K\langle 1, \frac{1}{t}, \frac{8t}{(t^2+1)^2} \rangle$

$6 = 3 + t + K$ $\quad K = 3 - t$

$4 = 2 + \ln t + \frac{K}{t}$ $\quad K = (2 - \ln t)t$

$t = 1$ $\quad K = 2$

≈ 16.8 $\quad = -13.8$