Let $|r(t)| = c$ $\qquad |r(t)| = \sqrt{r(t) \cdot r(t)}$

Prove $r(t) \cdot r'(t) = 0$.

$$|r(t)|^2 = c^2 = r(t) \cdot r(t)$$

$$\frac{d(c^2)}{dt} = 0 = r'(t) \cdot r(t) + r(t) r'(t)$$

$$0 = 2r'(t) \cdot r(t)$$

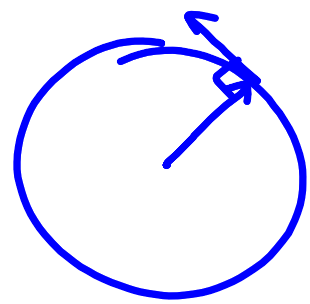
$$0 = r'(t) \cdot r(t)$$

$r(t) = \langle a\cos t, a\sin t \rangle$ ← position

$|r(t)| = a$

$r'(t) = \langle -a\sin t, a\cos t \rangle$ ← velocity

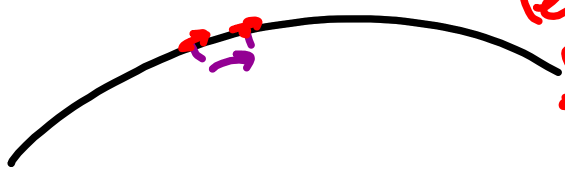
$r(t) \cdot r'(t) = 0$



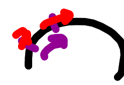
ex) $T \cdot T' = 0$

b/c $|T| = 1$

Curvature $= \left|\frac{\Delta T}{\Delta s}\right| = \left|\frac{dT}{ds}\right| \qquad \frac{dT}{dt}$



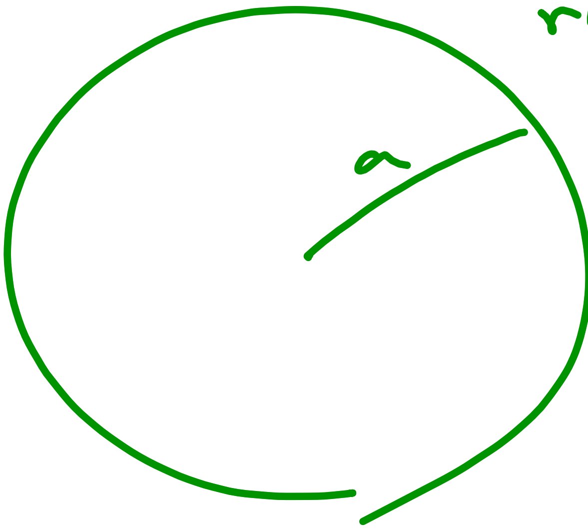
length of arc $\rightarrow s = \int |r'(t)|\,dt$



$$\frac{ds}{dt} = |r'(t)|$$

$$\frac{dT}{dt} = \frac{dT}{ds}\cdot\frac{ds}{dt}$$

$$\left|\frac{dT}{ds}\right| = \left|\frac{\frac{dT}{dt}}{\frac{ds}{dt}}\right| = \left|\frac{T'}{|r'(t)|}\right| = \frac{|T'|}{|r'(t)|}$$



$$r(t) = \langle a\cos t, a\sin t \rangle$$

$$K = \frac{|T'|}{|r'|} \qquad |r'| = a$$

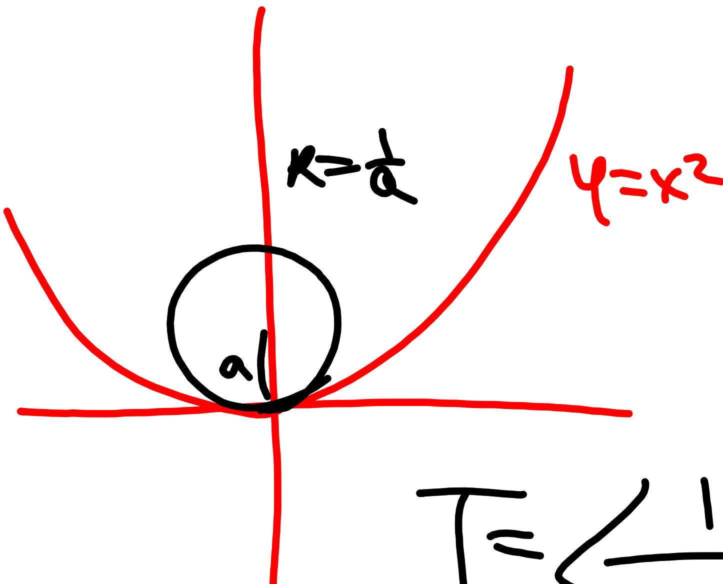
$$T = \frac{r'}{|r'|} = \frac{\langle -a\sin t, a\cos t \rangle}{a}$$

$$= \langle -\sin t, \cos t \rangle$$

$$K = \frac{|T'|}{|r'|} = \frac{1}{a}$$

$$T' = \langle -\cos t, -\sin t \rangle$$

$$|T'| = 1$$



$r(t) = \langle t, t^2 \rangle$ $\underline{\underline{(t=0)}}$

$r' = \langle 1, 2t \rangle$

$|r'| = \sqrt{1+4t^2}$

$= 1$

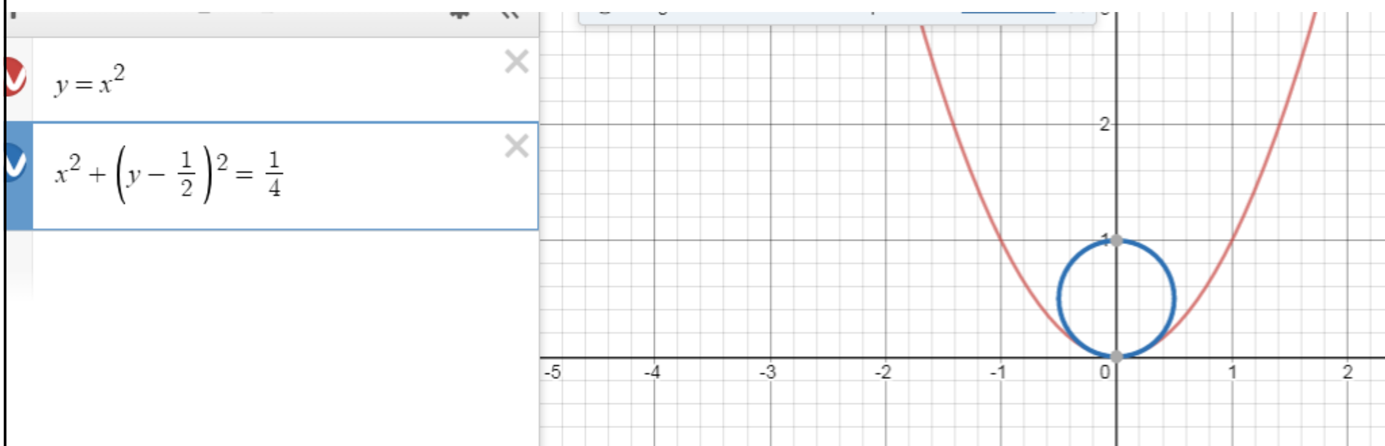
$$T = \left\langle \frac{1}{\sqrt{1+4t^2}}, \frac{2t}{\sqrt{1+4t^2}} \right\rangle$$

$$T' = \left\langle -\frac{1}{2}(1+4t^2)^{-\frac{3}{2}}(8t), \frac{2\sqrt{1+4t^2} - \frac{1 \cdot 8t}{2\sqrt{1+4t^2}}}{(1+4t^2)} \right\rangle$$

$\underline{\underline{t=0}}$ $\langle 0, 2 \rangle$ $|T'| = 2$

$$K = \frac{2}{1} = 2$$

$y = x^2$ $(x)^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$


$y = x^2$
$x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$

$r(t) = \langle t, t^2, t^3 \rangle$

Find the curvature at $t=0$.