27. At what points does the curve $\mathbf{r}(t) = t\,\mathbf{i} + (2t - t^2)\,\mathbf{k}$ intersect the paraboloid $z = x^2 + y^2$?

$= \langle t, 0, 2t - t^2 \rangle$

$2t - t^2 = t^2 + \cancel{0^2}$

$t^2 - t = 0$

$\therefore\ t = 0 \quad t = 1$

$\langle 0,0,0 \rangle \quad \langle 1,0,1 \rangle$

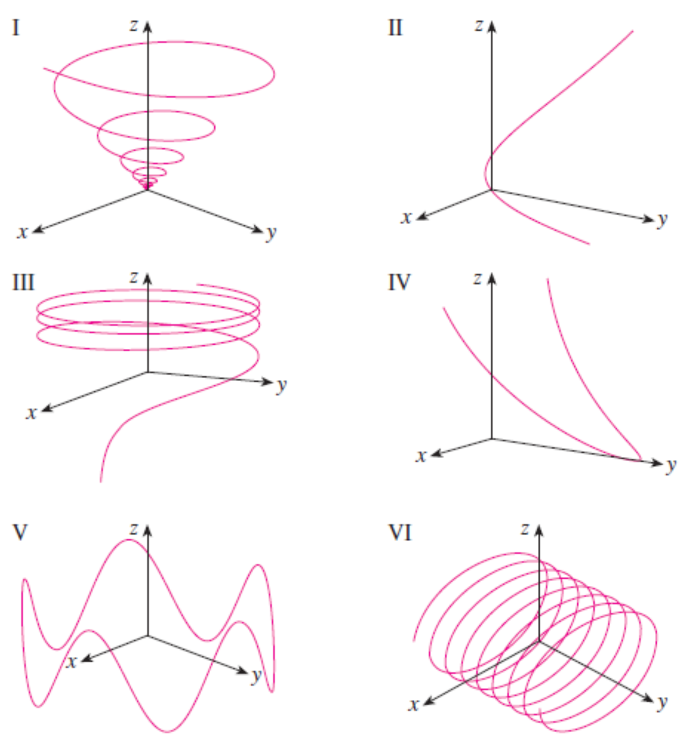
23. $x = \cos t,\quad y = \sin t,\quad \cancel{z = \sin 5t}$

$\boxed{\mathbf{21.}}$ $x = t,\quad y = 1/(1 + t^2),\quad z = t^2$

24. $x = \cos t,\quad y = \sin t,\quad z = \ln t$

$y = \dfrac{1}{1+z}$

$z = \dfrac{1}{y} - 1$



23–26 Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

23. $x = 1 + 2\sqrt{t}$, $y = t^3 - t$, $z = t^3 + t$; $(3, 0, 2)$

$r'(t) = \langle \frac{1}{\sqrt{t}}, 3t^2 - 1, 3t^2 + 1 \rangle, \ t = 1$

$\underset{t=1}{=} \langle 1, 2, 4 \rangle$

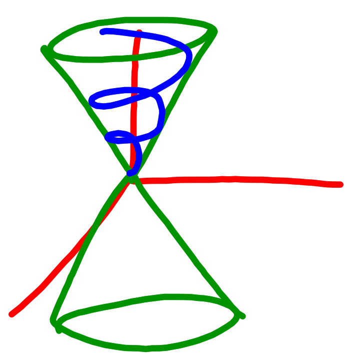
$r(t) = \langle 3, 0, 2 \rangle + t\langle 1, 2, 4 \rangle$

25. Show that the curve with parametric equations $x = t\cos t$, $y = t\sin t$, $z = t$ lies on the cone $z^2 = x^2 + y^2$, and use this fact to help sketch the curve.

$z = \sqrt{x^2 + y^2}$

$t^2 = (t\cos t)^2 + (t\sin t)^2$

$t^2 = t^2$ ✓

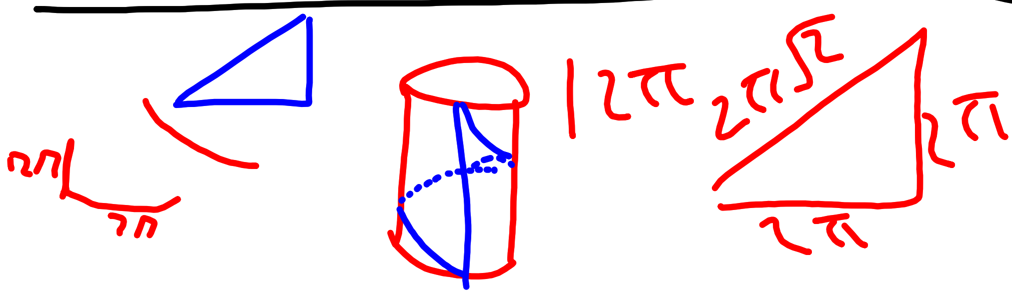


$r(t) = \langle \cos t, \sin t, 0 \rangle \quad 0 \le t \le 2\pi$

Find the length of the curve.: 2π

$r(t) = \langle \cos t, \sin t, t \rangle \quad 0 \le t \le 2\pi$

Find the length of the curve.



$$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2 + 1^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{2}\, dt = \sqrt{2}\, t \Big]_0^{2\pi} = 2\pi\sqrt{2}$$

$$L = \int_a^b |r'(t)|\, dt$$