$\boxed{1}$ **DEFINITION** If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **cross product** of $\mathbf{a}$ and $\mathbf{b}$ is the vector

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

$\langle a_1 \quad a_2 \quad a_3 \rangle$

$\langle b_1 \quad b_2 \quad b_3 \rangle$

$= i \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$

$\langle 3, 1, 0 \rangle \times \langle 2, 5, 0 \rangle$

$= \begin{vmatrix} i & j & k \\ 3 & 1 & 0 \\ 2 & 5 & 0 \end{vmatrix}$

$a \times b \perp a$ ~~or~~ (and) $a \times b \perp b$ $\Big\}$ $(a \times b) \cdot a = 0$

$= i \cdot 0 - j \cdot 0 + k \cdot 13 = \langle 0, 0, 13 \rangle$

$\langle 2, 5, 0 \rangle \times \langle 3, 1, 0 \rangle$

2 5 0

3 1 0

2 5

3 1

$\langle 0, 0, -13 \rangle$

$0 i \quad 0 - j \quad \cancel{+} k$

$-13k$

$2 - 15$

6 THEOREM If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ (so $0 \leqslant \theta \leqslant \pi$), then

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$$

ROOF From the definitions of the cross product and length of a vector, we have

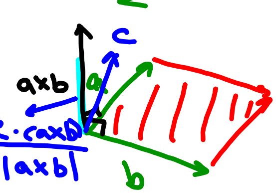
$$\begin{aligned}
|\mathbf{a} \times \mathbf{b}|^2 &= (a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2 \\
&= a_2^2b_3^2 - 2a_2a_3b_2b_3 + a_3^2b_2^2 + a_3^2b_1^2 - 2a_1a_3b_1b_3 + a_1^2b_3^2 \\
&\qquad + a_1^2b_2^2 - 2a_1a_2b_1b_2 + a_2^2b_1^2 \\
&= (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) - (a_1b_1 + a_2b_2 + a_3b_3)^2 \\
&= |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2 \\
&= |\mathbf{a}|^2|\mathbf{b}|^2 - |\mathbf{a}|^2|\mathbf{b}|^2\cos^2\theta \qquad \text{(by Theorem 12.3.3)} \\
&= |\mathbf{a}|^2|\mathbf{b}|^2(1 - \cos^2\theta) \\
&= |\mathbf{a}|^2|\mathbf{b}|^2\sin^2\theta
\end{aligned}$$

$|a \times b| = |a||b| \sin\theta$

$|a \times b| = 0$ if $a \parallel b$ (if $a = kb$)

$A_\Delta = \frac{1}{2} b h$

$= \frac{1}{2} ab \sin C$



$h = \dfrac{c \cdot (a \times b)}{|a \times b|}$

$|a \times b| =$ area of ▱ $= |b \cdot (a \times c)|$

$V = bA \cdot h$

$= |a \times b| \dfrac{c \cdot (a \times b)}{|a \times b|} = |c \cdot (a \times b)|$