HW Review

9. Determine whether the points lie on straight line.
   (a) $A(2, 4, 2)$, $B(3, 7, -2)$, $C(1, 3, 3)$
   (b) $D(0, -5, 5)$, $E(1, -2, 4)$, $F(3, 4, 2)$

① $\vec{AB} = \langle 1, 3, -4 \rangle \rightarrow |\vec{AB}| = \sqrt{26}$

$\vec{BC} = \langle -2, -4, 5 \rangle \rightarrow \sqrt{45}$

$\vec{AC} = \langle -1, -1, 1 \rangle$ $\sqrt{3}$

since $|\vec{AB}| + |\vec{BC}| \neq |\vec{AC}|$

A, B, & C are not collinear.

A. B. $|\vec{AB}| + |\vec{BC}| = |\vec{AC}|$

$\vec{AB} = k\vec{BC}$

② $\vec{DE} = \langle 1, 3, -1 \rangle$ $\sqrt{11}$

$\vec{EF} = \langle 2, 6, -2 \rangle$ $2\sqrt{11}$

$\vec{DF} = \langle 3, 9, -3 \rangle$ $3\sqrt{11}$

since $\vec{DE} = k\vec{EF}$, D, E, & F are collinear.

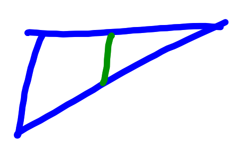
19. (a) Prove that the midpoint of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

$x_2 - \left(\frac{x_1 + x_2}{2}\right)$

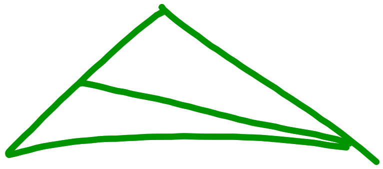
(b) Find the lengths of the medians of the triangle with vertices $A(1, 2, 3)$, $B(-2, 0, 5)$, and $C(4, 1, 5)$.

$\vec{P_1M} = \left\langle \frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2}, \frac{z_2 - z_1}{2} \right\rangle$



$\vec{MP_2} = \left\langle \frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2}, \frac{z_2 - z_1}{2} \right\rangle$

since $\vec{P_1M} = \vec{MP_2}$, M is the mid pt of $\overline{P_1P_2}$.



$A(1,2,3)$ $\quad M_{AB}(-.5,1,4)$

$B(-2,0,5)$ $\quad M_{BC}(1,.5,5)$

$C(4,1,5)$

$$CM_{AB} = \sqrt{4.5^2 + 0 + 1^2} = \frac{\sqrt{85}}{2}$$

$$AM_{BC} = \sqrt{0 + 1.5^2 + 2^2} = 2.5$$

Dot product.

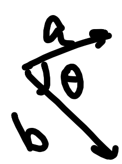
$$a = \langle a_1, a_2, a_3 \rangle$$

$$b = \langle b_1, b_2, b_3 \rangle$$

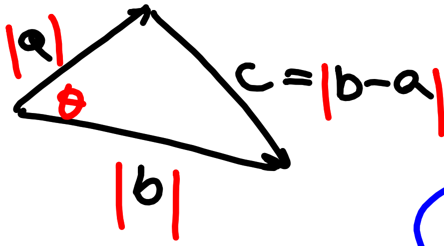
$$a \cdot b = a_1b_1 + a_2b_2 + a_3b_3$$

$$a \cdot b = b \cdot a$$

$$a \cdot b = |a||b| \cos\theta$$



$a \cdot b = |a||b|\cos\theta$



$a = \langle a_1, a_2, \cdots, a_n \rangle$

$b = \langle b_1, b_2, \cdots, b_n \rangle$

$|b-a|^2 = |a|^2 + |b|^2 - 2|a||b|\cos\theta$

$(b_1 - a_1)^2$ $\quad a_1 \cdot a_1 + a_2 a_2 \cdots$

$b_1^2 - 2b_1a_1 + a_1^2$ $\quad b_1 b_1$

$-2b_1a_1 \cdots = -2|a||b|\cos\theta$

ba

$a \cdot b = |a||b|\cos\theta$

$-2b_1a_1 - 2b_2a_2 - \cdots$

$-2(b\cdot a) = -2a\cdot b$

$|b-a|^2$

$= |\langle b_1 - a_1, b_2 - a_2, \cdots, b_n - a_n\rangle|^2$

$= (b_1-a_1)^2 + (b_2-a_2)^2 + \cdots + (b_n-a_n)^2$

$= \cancel{b_1^2 + b_2^2 + \cdots + b_n^2} \; \cancel{+ a_1^2 + a_2^2 + \cdots + a_n^2}$

$-2b_1a_1 - 2b_2a_2 - \cdots - 2b_na_n = \cancel{|a|^2} + \cancel{|b|^2} - 2|a||b|\cos\theta$

$b_1a_1 + b_2a_2 + \cdots + b_na_n$

$= b\cdot a = a\cdot b = |a||b|\cos\theta$