2 Percent

EXAMPLE 1: Jacob got 50% of the questions correct on a 30-question test and 90% on a 50 question test. What percent of all questions did Jacob get correct?

First, let's find the total number of questions he got correct:

$$50\% \times 30 = \frac{1}{2} \times 30 = 15$$

$$90\% \times 50 = \frac{9}{10} \times 50 = 45$$

So he got $15 + 45 = 60$ questions correct out of a total of $30 + 50 = 80$ questions. $\frac{60}{80} = \frac{3}{4} = \boxed{75}\%$

EXAMPLE 2: A record of driving violations by type and vehicle is shown below.

	Violation Type			
	Speeding	Stop Sign	Parking	Total
Truck	68	39	17	124
Car	83	51	26	160
Total	151	90	43	284

PART 1: Which of the following is closest to the percent of recorded parking violations that were committed by trucks?

A) 6% B) 14% C) 40% D) 65%

PART 2: If the data were used to estimate driving violation information about 2,000 total violations in a certain state, which of the following is the best estimate of the number of speeding violations committed by cars in the state?

A) 479 B) 585 C) 1063 D) 1099

Part 1 Solution:

$$\frac{\text{Truck Parking Violations}}{\text{Total Parking Violations}} = \frac{17}{43} = 0.3953 \approx 40\%$$

Answer $\boxed{(C)}$.

Part 2 Solution: The SAT will often ask you to estimate or predict certain information based on a smaller sample size. In these questions, take what you learn from the smaller size and simply apply it to the larger population. From the sample size, $\frac{\text{Car Speeding Violations}}{\text{Total Parking Violations}} = \frac{83}{284}$. Now we can apply this same proportion to the state total of 2,000:

$$\frac{83}{284} \times 2000 \approx 585$$

Answer $\boxed{(B)}$.

EXAMPLE 3: The price of a dress is increased by 20%, then decreased by 40%, then increased by 25%. The final price is what percent of the original price?

Here's the technique for dealing with these "series of percent change" questions. Let the original price be p. When p is increased by 20%, you multiply by 1.20 because it's the original price plus 20%. When it's decreased by 40%, you multiply by .60 because 60% is what's left after you take away 40%. Our final price is then

$$p \times 1.20 \times .60 \times 1.25 = .90p$$

The final price is $\boxed{90\%}$ of the original price.

Example 3 shows the MOST IMPORTANT percent concept by far on the SAT. Never ever calculate the prices at each step. String all the changes together to get the end result.

It's important to know why this works. Imagine again that the original price is p and we want to increase it by 20%. Normally, we would just take p and add 20% of it on top:

$$p + .20p$$

But realize that

$$p + .20p = p(1 + .20) = 1.20p$$

And now we want to decrease this new price by 40%:

$$1.20p - (.40)(1.20p) = (1.20p)(1 - .40) = (1.20p)(0.60) = (1.20)(0.60)p$$

which proves we can calculate the final price directly through this technique. Now we're set up to tackle the inevitable compound interest questions on the SAT.

EXAMPLE 4: Jonas has a savings account that earns 3 percent interest compounded annually. Her initial deposit was \$1000. Which of the following expressions gives the value of the account after 10 years?

A) $1000(1.30)^{10}$ B) $1000 + 30(10)$ C) $1000(1.03)(10)$ D) $1000(1.03)^{10}$

A 3 percent interest rate compounded annually means he earns 3 percent on the account once a year. Keep in mind that this isn't just 3% on the original amount of \$1000. This is 3% of whatever's in the account at the time, including any interest that he's already earned in previous years. This is the meaning of **compound interest**. So if we're in year 5, he would earn 3% on the original \$1000 and 3% on the total interest deposited in years 1 through 4.

If we try to calculate the total after each and every year, this problem would take forever. Let's take what we learned from Example 3 and apply it here:

Year 1 total: $1000(1.03) = 1000(1.03)^1$
Year 2 total: $1000(1.03)(1.03) = 1000(1.03)^2$
Year 3 total: $1000(1.03)(1.03)(1.03) = 1000(1.03)^3$
Year 4 total: $1000(1.03)(1.03)(1.03)(1.03) = 1000(1.03)^4$

See the pattern? Each year is an increase of 3% so it's just 1.03 times whatever the value was last year. Note that we're not doing any calculations out. Think of it as the price of a dress being increased by 3% ten times.

Therefore, the Year 10 total is $1000(1.03)^{10}$, answer $\boxed{(D)}$.

Most of these compound interest questions can be modeled by the equation $A = P(1 + r)^t$, where A is the total amount accumulated, P is the principal or the initial amount, r is the interest rate, and t is the number of times interest is received.

EXAMPLE 5: Jay puts an initial deposit of \$400 into a bank account that earns 5 percent interest each year, compounded semiannually. Which of the following equations gives the total dollar amount, A, in the account after t years?

A) $A = 400(1 + 0.05t)$ B) $A = 400(1 + 0.1t)$ C) $A = 400(1.05)^t$ D) $A = 400(1.025)^{2t}$

The interest is compounded *semiannually*. That means twice a year. So interest is received $2t$ times. However, we don't receive a full 5% each time interest is received. The 5% interest rate is a yearly figure. We have to divide it by 2 to get the semiannual rate: 2.5%. The answer is $\boxed{(D)}$.

Note that semiannual compounding is better than annual compounding. Why? With annual compounding, you just get 5% on the initial amount after one year. That's just like 2.5% on the initial amount and then another 2.5% on the initial amount. But with semiannual compounding, you get 2.5% on the initial amount and then you get 2.5% on the mid-year amount, which is greater than the initial amount because it includes the first interest payment. Because you've already earned interest before the end of the year, you get a little extra. This might not seem like a lot, but over many years, it can make a huge difference. The more times interest is compounded, the more money you accumulate.

If interest is compounded more than once a year, the previous formula can be generalized to

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A is the total amount accumulated, P is the principal or initial amount, r is the interest rate, t is the number of years, and n is the number of times the interest is compounded each year. You don't need to memorize these formulas if you understand the underlying math.

Now that we've shown you how to handle compound interest questions, let's take a step back to bring up **simple interest**. While compound interest lets you earn interest on interest you've earned, simple interest means you get the same amount each time. Interest is earned only on the original amount, not on any interest you've earned.

EXAMPLE 6: An investor decides to offer a business owner a \$20,000 loan at simple interest of 5% per year. Which of the following functions gives the total amount, A, in dollars, the investor will receive when the loan is repaid after t years?

A) $A = 20,000(1.05)^t$ B) $A = 20,000(1 + 0.05^t)$ C) $A = 20,000(1.05t)$ D) $A = 20,000(1 + 0.05t)$

At a simple interest of 5%, the investor will receive $20,000(0.05)$ in interest each year. That amount does not change because the 5% always applies to the original \$20,000 under simple interest. So after t years, he will receive a total of $20,000(0.05)t$ in interest.

The amount he will be repaid after t years is then

$$\begin{aligned} A &= \text{Original amount} + \text{Total interest} \\ &= 20,000 + 20,000(0.05)t \\ &= 20,000(1 + 0.05t) \end{aligned}$$

Notice how we factored out the 20,000 in the last step. The answer is $\boxed{(D)}$. The answer would have been (A) under compound interest.

For simple interest, the formula is

$$A = P(1 + rt)$$

where A is the total amount accumulated, P is the principal or initial amount, r is the interest rate, and t is the number of times interest is earned (typically the number of years).

EXAMPLE 7: This year, the chickens on a farm laid 30% less eggs than they did last year. If they laid 3,500 eggs this year, how many did they lay last year?

$$\begin{aligned} \text{This Year} &= (.70)(\text{Last Year}) \\ 3,500 &= (.70)(\text{Last Year}) \\ \boxed{5,000} &= \text{Last Year} \end{aligned}$$

Percent change (a.k.a. percent increase/decrease) is calculated as follows:

$$\%\text{ change} = \frac{\text{new value} - \text{old value}}{\text{old value}} \times 100$$

For example, if the price of a dress starts out at 80 dollars and rises to 90 dollars, the percent change is:

$$\frac{90-80}{80} \times 100 = 12.5\%$$

If percent change is positive, it's a percent increase. Negative? Percent decrease. It's important to remember that percent change is always based on the original value.

EXAMPLE 8: In a particular store, the number of TVs sold the week of Black Friday was 685. The number of TVs sold the following week was 500. TV sales the week following Black Friday were what percent less than TV sales the week of Black Friday (rounded to the nearest percent)?

A) 17% B) 27% C) 37% D) 47%

$$\frac{500-685}{685} \approx -0.27$$

We put the difference over 685, NOT 500. Answer $\boxed{(B)}$.

EXAMPLE 9: In a particular store, the number of computers sold the week of Black Friday was 470. The number of computers sold the previous week was 320. Which of the following best approximates the percent increase in computer sales from the previous week to the week of Black Friday?

A) 17% B) 27% C) 37% D) 47%

$$\frac{470-320}{320} \approx 0.47$$

This time, the week of Black Friday is not the "original" basis for the percent change. We put the difference over the previous week's number, 320. The answer is $\boxed{(D)}$.

A few more examples involving percent:

EXAMPLE 10: The number of students at a school decreased 20% from 2010 to 2011. If the number of students enrolled in 2011 was k, which of the following expresses the number of students enrolled in 2010 in terms of k?

A) $0.75k$ B) $1.20k$ C) $1.25k$ D) $1.5k$

The answer is NOT $1.20k$. Percent change is based off of the original value (from 2010) and not the new value. Let x be the number of students in 2010,

$$.80x = k$$

$$x = 1.25k$$

Therefore, there were 25% more students in 2010 than in 2011. Answer $\boxed{(C)}$.

EXAMPLE 11: Among 10th graders at a school, 40% of the students are Red Sox fans. Among those Red Sox fans, 20% are also Celtics fans. What percent of the 10th graders at the school are both Red Sox fans and Celtics fans?

We don't know the number of 10th graders at the school so let's suppose that it's 100.

$$\text{Red Sox fans} = 40\% \text{ of } 100 = 40$$

$$\text{Celtics \& Red Sox fans} = 20\% \text{ of } 40 = 8$$

The answer is then $\dfrac{8}{100} = \boxed{8\%}$

A common strategy in percent questions is to make up a number to represent the total, typically 100.

CHAPTER EXERCISE: Answers for this chapter start on page 256.

A calculator is allowed on the following questions.

1

If x is 50% larger than z, and y is 20% larger than z, then x is what percent larger than y?

A) 15%
B) 20%
C) 25%
D) 30%

2

Veronica has a bank account that earns $m\%$ interest compounded annually. If she opened the account with \$200, the expression $\$200(x)^t$ represents the amount in the account after t years. Which of the following gives x in terms of m?

A) $1 + .01m$
B) $1 + m$
C) $1 - m$
D) $1 + 100m$

3

In a survey of 400 seniors, x percent said that they plan on majoring in physics. One university has used this data to estimate the number of physics majors it expects for its entering class of 3,300 students. If the university expects 66 physics majors, what is the value of x?

Questions 4-7 refer to the following information.

The table below shows the number of box spring and mattress units sold over four weeks at a bedding store.

Week	1	2	3	4	Total
Box Springs	38	42	53	34	167
Mattresses	47	61	68	43	219
Total	85	103	121	77	386

4

Which week accounted for approximately 32% of all the box spring units sold?

A) Week 1
B) Week 2
C) Week 3
D) Week 4

5

Approximately what percentage of all units sold came from week 2?

A) 15.8%
B) 26.7%
C) 31.3%
D) 47.0%

6

Mattresses accounted for approximately what percentage of all units sold during week 1?

A) 22.0%
B) 32.5%
C) 44.7%
D) 55.3%

7

What was the approximate percent decrease in the number of mattresses sold from week 3 to week 4?

A) 37%
B) 41%
C) 46%
D) 58%

8

The discount price of a book is 20% less than the retail price. James manages to purchase the book at 30% off the discount price at a special book sale. What percent of the retail price did James pay?

A) 42%
B) 48%
C) 50%
D) 56%

9

Each day, Robert eats 40% of the pistachios left in his jar at that time. At the end of the second day, 27 pistachios remain. How many pistachios were in the jar at the start of the first day?

A) 75
B) 80
C) 85
D) 95

10

Joanne bought a doll at a 10 percent discount off the original price of $105.82. However, she had to pay a sales tax of x% on the discounted price. If the total amount she paid for the doll was $100, what is the value of x?

A) 2
B) 3
C) 4
D) 5

11

In 2010, the number of houses built in Town A was 25 percent greater than the number of houses built in Town B. If 70 houses were built in Town A during 2010, how many were built in Town B?

12

Over a two week span, John ate 20 pounds of chicken wings and 15 pounds of hot dogs. Kyle ate 20 percent more chicken wings and 40 percent more hot dogs. Considering only chicken wings and hot dogs, Kyle ate approximately x percent more food, by weight, than John. What is x (rounded to the nearest percent)?

A) 25
B) 27
C) 29
D) 30

13

Jane is playing a board game in which she must collect as many cards as possible. On her first turn, she loses 18 percent of her cards. On the second turn, she increases her card count by 36 percent. If her final card count after these two turns is n, which of the following represents her starting card count in terms of n?

A) $\frac{n}{(1.18)(0.64)}$

B) $(1.18)(0.64)n$

C) $\frac{n}{(1.36)(0.82)}$

D) $(0.82)(1.36)n$

14

Due to deforestation, researchers expect the deer population to decline by 6 percent every year. If the current deer population is 12,000, what is the approximate expected population size 10 years from now?

A) 4800

B) 6460

C) 7240

D) 7980

15

Kyle bought a $2,000 government bond that yields 6% in simple interest each year. Which of the following equations gives the total amount A, in dollars, Kyle will receive when he sells the bond after t years?

A) $A = 2,000(1 + .06)t$

B) $A = 2,000(1 + 0.06t)$

C) $A = 2,000(1 + 0.06)^t$

D) $A = 2,000(1 + 0.06^t)$

16

A small clothing store sells 3 different types of accessories: 20% are scarves, 60% are ties, and the other 40 accessories are belts. If half of the ties are replaced with scarves, how many scarves will the store have?

17

Daniel has $1000 in a checking account and $3000 in a savings account. The checking account earns him 1 percent interest compounded annually. The savings account earns him 6 percent interest compounded annually. Assuming he leaves both these accounts alone, which of the following represents how much more interest Daniel will have earned from the savings account than from the checking account after 5 years?

A) $3,000(1.06)^5 - 1,000(1.01)^5$

B) $3,000(1.06)(5) - 1,000(1.01)(5)$

C) $(3,000(1.06)^5 - 3,000) - (1,000(1.01)^5 - 1,000)$

D) $(3,000(1.06)(5) - 3,000) - (1,000(1.01)(5) - 1,000)$

18

Kristen opens a bank account that earns 4% interest each year, compounded once every two years. If she opened the account with k dollars, which of the following expressions represents the total amount in the account after t years?

A) $k(1.04)^{2t}$

B) $k(1.04)^{\frac{t}{2}}$

C) $k(1.08)^t$

D) $k(1.08)^{\frac{t}{2}}$

Chapter 2: Percent

CHAPTER EXERCISE:

1. $\boxed{C}$ Let $z = 100$. Then $x = 1.50(100) = 150$ and $y = 1.20(100) = 120$. x is

$$\frac{150 - 120}{120} = \frac{30}{120} = 25\%$$

larger than y.

2. $\boxed{A}$ Each year, Veronica keeps whatever she has in her account plus the interest on that amount. Because m is a percentage, we can convert it to a decimal by dividing it by 100, giving us $0.01m$. Therefore, $x = 1 + 0.01m$.

3. $\boxed{2}$ Note that the sample size of 400 is irrelevant information. To make things easier, we'll let x be a decimal for now and convert it to a percentage later,

$$\begin{aligned} 3,300x &= 66 \\ x &= 0.02 = 2\% \end{aligned}$$

4. $\boxed{C}$ Week 3 accounted for $\frac{53}{167} \approx 0.32 = 32\%$ of the total box spring units sold.

5. $\boxed{B}$ $\frac{103}{386} \approx 0.267 = 26.7\%$

6. $\boxed{D}$ $\frac{47}{85} \approx 0.553 = 55.3\%$

7. $\boxed{A}$ $\frac{43 - 68}{68} \approx -0.37 = 37\%$ decrease.

8. $\boxed{D}$ Let the original price of the book be \$100. Then James bought the book at $100(1 - 0.20)(1 - 0.30) = 100(0.80)(0.70) =$ \$56, which is $\frac{56}{100} = 56\%$ of the original price.

9. $\boxed{A}$ Let x be the number of pistachios at the start. At the end of each day, what's left is $1 - 0.40 = 0.60$ of the day's starting amount. Over two days,

$$\begin{aligned} x(0.60)(0.60) &= 27 \\ 0.36x &= 27 \\ x &= 75 \end{aligned}$$

10. $\boxed{D}$ Let x be the sales tax (as a decimal for now). We'll convert it to a percent at the end.

$$\begin{aligned} 105.82(.90)(1 + x) &= 100 \\ 1 + x &= \frac{100}{(105.82)(.90)} \\ x &= \frac{100}{(105.82)(.90)} - 1 = 0.05 = 5\% \end{aligned}$$

11. $\boxed{56}$

$$\begin{aligned} A &= (1.25)(B) \\ 70 &= (1.25)(B) \\ 56 &= B \end{aligned}$$

12. $\boxed{C}$ Kyle ate $20(1.20) = 24$ pounds of chicken wings and $15(1.40) = 21$ pounds of hot dogs. That's a total of $24 + 21 = 45$ pounds of food. John had $20 + 15 = 35$ pounds of food. The percent increase from John to Kyle is

$$\frac{45 - 35}{35} \approx .29 = 29\%$$

13. $\boxed{C}$ Let her starting card count be x. A loss of 18 percent reduces her total to $(0.82)x$. From there, an increase of 36 percent gets the total to $(1.36)(0.82)x$. Now,

$$\begin{aligned} (1.36)(0.82)x &= n \\ x &= \frac{n}{(1.36)(0.82)} \end{aligned}$$

14. [B] $12,000(0.94)^{10} \approx 6,460$.

15. [B] Simple interest means that Kyle will receive the same amount each year based on his initial investment of \$2,000. He'll receive $2,000(0.06)$ in interest each year for a total of $2,000(0.06)t$ after t years. He'll also get back his initial investment. Therefore, the total amount he receives after t years is

$$2,000 + 2,000(0.06)t = 2,000(1 + 0.06t)$$

16. [100] Since scarves and ties make up 80% of the accessories, the 40 belts must account for 20%. Letting the total number of accessories be x,

$$20\% \text{ of } x = 40$$

$$\frac{1}{5}x = 40$$

$$x = 200$$

There are 200 accessories in the store. Hopefully you're able to get this without having to make an equation, but there's no harm in a little algebra! Now we can determine that there are $\frac{1}{5} \times 200 = 40$ scarves and $\frac{3}{5} \times 200 = 120$ ties. Half of the 120 ties (60 ties) are replaced with scarves, so the store will end up with $40 + 60 = 100$ scarves.

17. [C] The total amount in the savings account after 5 years will be $3,000(1.06)^5$, but the interest earned will be $3,000(1.06)^5 - 3,000$. The total amount in the checking account after 5 years will be $1,000(1.01)^5$, but the interest earned will be $1,000(1.01)^5 - 1,000$. With a larger initial deposit and a higher interest rate, it's obvious the savings account will have earned more interest. The difference in earned interest will be $(3,000(1.06)^5 - 3,000) - (1,000(1.01)^5 - 1,000)$.

18. [D] Compounded once every 2 years, interest is earned $\frac{t}{2}$ times in t years. The annual interest rate of 4% must be doubled to get the rate earned over a 2 year span. Therefore, the correct expression is $k(1.08)^{\frac{t}{2}}$.