Limits and Continuity

Brief Review

Limit – intended height (y-value) of the function.

Properties: add, subtract, divide, multiply, multiply constant and raise to any power.

Techniques to Evaluation:

- Direct Substitution – plug the x-value in…if you get a number you are done…if you get an indeterminate form….

  1.) Try to factor the expression. Cancel common factors and try direct substitution again.

  2.) Try tables or graphs….try plugging in a number close to the x-value to the right and the left.

  3.) If you are in BC Calculus try L'Hopital's Rule or a logarithm.

One sided limits:

$$\lim_{x \to c^+} f(x) \quad is\ a\ limit\ from\ the\ RIGHT$$

$$\lim_{x \to c^-} f(x) \quad is\ a\ limit\ from\ the\ LEFT$$

Limits that approach infinity:

If it's a rational function….take the largest term on the top and bottom and simplify and then take the limit.

Remember: 1/small = BIG (infinity) 1/BIG = SMALL(zero) …..and it doesn't matter if that 1 is a 4 or a 10 or a - 3.

CONTINUITY:

1.) Function value must exist.

2.) Limit must exist.

3.) Function value must equal the limit,

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1. $\lim_{x\to\infty}\frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

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5. $\lim_{x\to 0}\frac{5x^4+8x^2}{3x^4-16x^2}$ is

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}+1$ (E) nonexistent

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$$f(x)=\begin{cases}\frac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$.

II. f is continuous at $x=2$.

III. f is differentiable at $x=2$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

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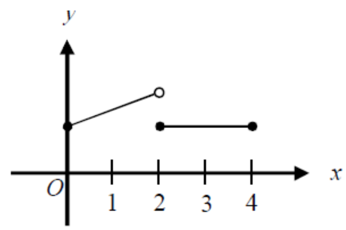

Graph of f

77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

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89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

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3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x \to 2} f(x) = \infty$

(E) $\lim_{x \to \infty} f(x) = 2$

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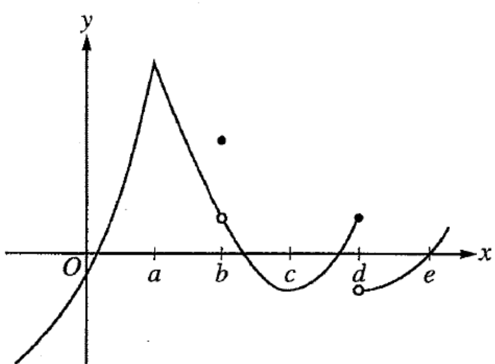
6. $$\lim_{x \to \infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$$

(A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

20. Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists.

II. f is continuous at $x = 3$.

III. f is differentiable at $x = 3$.

(A) None

(B) I only

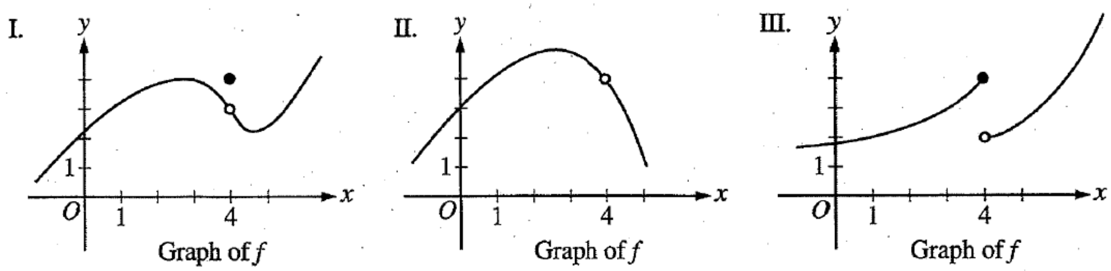
(C) II only

(D) I and II only

(E) I, II, and III

Calculator Active – 2003

79. For which of the following does $\lim_{x \to 4} f(x)$ exist?



(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

Free Response 2011 #6 Non-Calculator Active

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

Free Response 2011B #2 Calculator Active

2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

Free Response 2008 #6 Non-Calculator Active

6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

(d) Find $\lim_{x \to 0^+} f(x)$.

Free Response 2003 #6 Non-Calculator Active

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

Free Response Practice

Given the function $f(x) = \dfrac{x^3 + 2x^2 - 3x}{3x^2 + 3x - 6}$.

(a) What are the zeros of $f(x)$?

(b) What are the vertical asymptotes of $f(x)$?

(c) The end behavior model of $f(x)$ is the function $g(x)$. What is $g(x)$?

(d) What is $\lim_{x \to \infty} f(x)$? What is $\lim_{x \to \infty} \dfrac{f(x)}{g(x)}$?