21. B $f'(x)=2x-2e^{-2x}$, $f'(0)=-2$, so f is decreasing

22. E $\ln e^{2x}=2x \Rightarrow \frac{d}{dx}\left(\ln e^{2x}\right)=\frac{d}{dx}(2x)=2$

23. C $\int_0^2 e^{2x}\,dx=\frac{1}{2}e^{2x}\Big|_0^2=\frac{1}{2}\left(e^4-1\right)$

24. C $y=\ln\sin x$, $y'=\frac{\cos x}{\sin x}=\cot x$

25. A $\int_m^{2m}\frac{1}{x}dx=\ln x\Big|_m^{2m}=\ln(2m)-\ln(m)=\ln 2$ so the area is independent of m.

26. C $\int_0^1\sqrt{x^2-2x+1}\,dx=\int_0^1|x-1|\,dx=\int_0^1-(x-1)\,dx=-\frac{1}{2}(x-1)^2\Big|_0^1=\frac{1}{2}$

Alternatively, the graph of the region is a right triangle with vertices at (0,0), (0,1), and (1,0). The area is $\frac{1}{2}$.

27. C $\int\tan x\,dx=\int\frac{\sin x}{\cos x}dx=-\ln|\cos x|+C=\ln|\sec x|+C$

28. C $\sqrt{3}\cos x + 3\sin x$ can be thought of as the expansion of $\sin(x+y)$. Since $\sqrt{3}$ and 3 are too large for values of $\sin y$ and $\cos y$, multiply and divide by the result of the Pythagorean Theorem used on those values, i.e. $2\sqrt{3}$. Then

$$\sqrt{3}\cos x + 3\sin x = 2\sqrt{3}\left(\frac{\sqrt{3}}{2\sqrt{3}}\cos x + \frac{3}{2\sqrt{3}}\sin x\right) = 2\sqrt{3}\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$$
$$= 2\sqrt{3}(\sin y\cos x + \cos y\sin x) = 2\sqrt{3}\sin(y+x)$$

where $y = \sin^{-1}\left(\frac{1}{2}\right)$. The amplitude is $2\sqrt{3}$.

Alternatively, the function $f(x)$ is periodic with period 2π. $f'(x) = -\sqrt{3}\sin x + 3\cos x = 0$ when $\tan x = \sqrt{3}$. The solutions over one period are $x = \frac{\pi}{3}, \frac{4\pi}{3}$. Then $f\left(\frac{\pi}{3}\right) = 2\sqrt{3}$ and $f\left(\frac{4\pi}{3}\right) = -2\sqrt{3}$. So the amplitude is $2\sqrt{3}$.

29. A $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx = \ln(\sin x)\Big|_{\pi/4}^{\pi/2} = \ln 1 - \ln\frac{1}{\sqrt{2}} = \ln\sqrt{2}$

30. E Because f is continuous for all x, the Intermediate Value Theorem implies that the graph of f must intersect the x-axis. The graph must also intersect the y-axis since f is defined for all x, in particular, at $x = 0$.

31. C $\frac{dy}{dx} = -y \Rightarrow y = ce^{-x}$ and $1 = ce^{-1} \Rightarrow c = e;\ y = e\cdot e^{-x} = e^{1-x}$

32. B If $a < 0$ then $\lim_{x\to-\infty} y = \infty$ and $\lim_{x\to\infty} y = -\infty$ which would mean that there is at least one root.
If $a > 0$ then $\lim_{x\to-\infty} y = -\infty$ and $\lim_{x\to\infty} y = \infty$ which would mean that there is at least one root.
In both cases the equation has at least one root.

33. A $\frac{1}{3}\int_{-1}^{2} 3t^3 - t^2\,dt = \frac{1}{3}\left(\frac{3}{4}t^4 - \frac{1}{3}t^3\right)\Big|_{-1}^{2} = \frac{1}{3}\left(\left(12-\frac{8}{3}\right)-\left(\frac{3}{4}+\frac{1}{3}\right)\right) = \frac{11}{4}$

34. D $y' = -\frac{1}{x^2}$, so the desired curve satisfies $y' = x^2 \Rightarrow y = \frac{1}{3}x^3 + C$

35. A $a(t)=24t^2$, $v(t)=8t^3+C$ and $v(0)=0 \Rightarrow C=0$. The particle is always moving to the right, so distance = $\int_0^2 8t^3 dt = 2t^4\Big|_0^2 = 32$.

36. B $y=\sqrt{4+\sin x}$, $y(0)=2$, $y'(0)=\dfrac{\cos 0}{2\sqrt{4+\sin 0}}=\dfrac{1}{4}$. The linear approximation to y is $L(x)=2+\dfrac{1}{4}x$. $L(1.2)=2+\dfrac{1}{4}(1.2)=2.03$

37. D All options have the same value at $x=0$. We want the one that has the same first and second derivatives at $x=0$ as $y=\cos 2x$: $y'(0)=-2\sin 2x\Big|_{x=0}=0$ and $y''(0)=-4\cos 2x\Big|_{x=0}=-4$.

For $y=1-2x^2$, $y'(0)=-4x\Big|_{x=0}=0$ and $y''(0)=-4$ and no other option works.

38. C $\int \dfrac{x^2}{e^{x^3}}dx = -\dfrac{1}{3}\int e^{-x^3}(-3x^2dx) = -\dfrac{1}{3}e^{-x^3}+C = -\dfrac{1}{3e^{-x^3}}+C$

39. D $x=e \Rightarrow v=1,\ u=0,\ y=0;\ \dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dv}\cdot\dfrac{dv}{dx}=\left(\sec^2 u\right)\left(1+\dfrac{1}{v^2}\right)\left(\dfrac{1}{x}\right)=(1)(2)\left(e^{-1}\right)=\dfrac{2}{e}$

40. E One solution technique is to evaluate each integral and note that the value is $\dfrac{1}{n+1}$ for each.

Another technique is to use the substitution $u=1-x$; $\int_0^1(1-x)^n\,dx=\int_1^0 u^n(-du)=\int_0^1 u^n\,du$.

Integrals do not depend on the variable that is used and so $\int_0^1 u^n\,du$ is the same as $\int_0^1 x^n\,dx$.

41. D $\int_{-1}^3 f(x)\,dx=\int_{-1}^2\left(8-x^2\right)dx+\int_2^3 x^2dx=\left(8x-\dfrac{1}{3}x^3\right)\Big|_{-1}^2+\dfrac{1}{3}x^3\Big|_2^3=27\tfrac{1}{3}$

42. D $y=x^3-3x^2+k$, $y'=3x^2-6x=3x(x-2)$. So f has a relative maximum at $(0,k)$ and a relative minimum at $(2,k-4)$. There will be 3 distinct x-intercepts if the maximum and minimum are on the opposite sides of the x-axis. We want $k-4<0<k \Rightarrow 0<k<4$.

44. C Since $\cos 2A = 2\cos^2 A - 1$, we have $3 - 2\cos^2 \frac{\pi x}{3} = 3 - (1 + \cos \frac{2\pi x}{3})$ and the latter expression has period $\frac{2\pi}{\left(\frac{2\pi}{3}\right)} = 3$

45. D Let $y = f(x^3)$. We want y'' where $f'(x) = g(x)$ and $f''(x) = g'(x) = f(x^2)$

$$
\begin{aligned}
y &= f(x^3) \\
y' &= f'(x^3) \cdot 3x^2 \\
y'' &= 3x^2 \left(f''(x^3) \cdot 3x^2 \right) + f'(x^3) \cdot 6x \\
&= 9x^4 f''(x^3) + 6x\, f'(x^3) = 9x^4 f((x^3)^2) + 6x\, g(x^3) = 9x^4 f(x^6) + 6x\, g(x^3)
\end{aligned}
$$