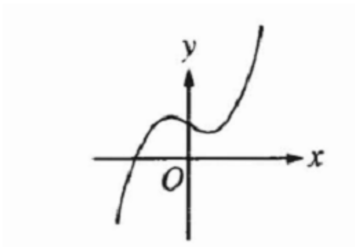
Relating f, f’, and f”

AP Calculus

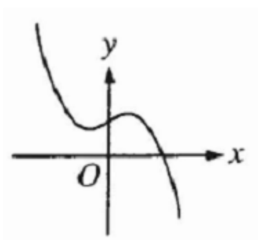
Multiple Choice Questions

1. **1998 #6 (BC) - No Calc:**

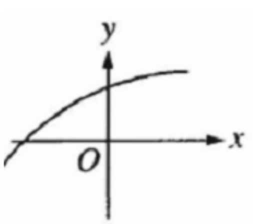


The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

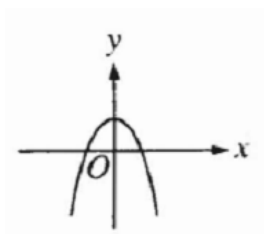
a.



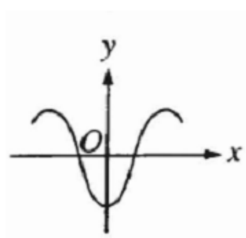
b.



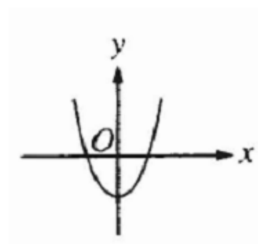
c.



d.



e.



2. **1998 #1 (BC) - No Calc:** What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

a. $-3 < x < 1$

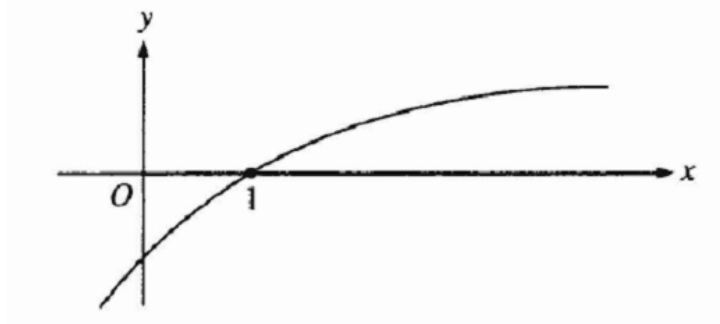
b. $-1 < x < 1$

c. $x < -3$ or $x > 1$

d. $x < -1$ or $x > 3$

e. All real numbers

3. **1998 #17 (BC) - No Calc:**



The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

a. $f(1) < f'(1) < f''(1)$

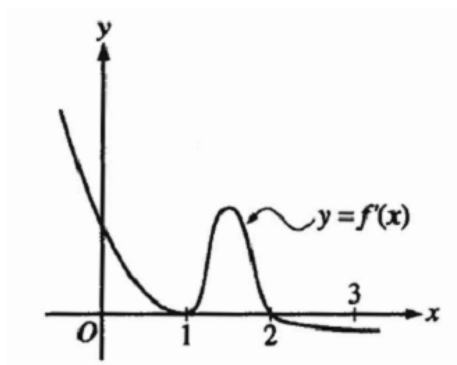
b. $f(1) < f''(1) < f'(1)$

c. $f'(1) < f(1) < f''(1)$

d. $f''(1) < f(1) < f'(1)$

e. $f''(1) < f'(1) < f(1)$

4. **2003 #90 (BC) - Calc OK:**



The graph of f', the derivative of the function f, is shown above. If $f(0) = 0$, which of the following must be true?

I. $f(0) > f(1)$
II. $f(2) > f(1)$
III. $f(1) > f(3)$

a. I only

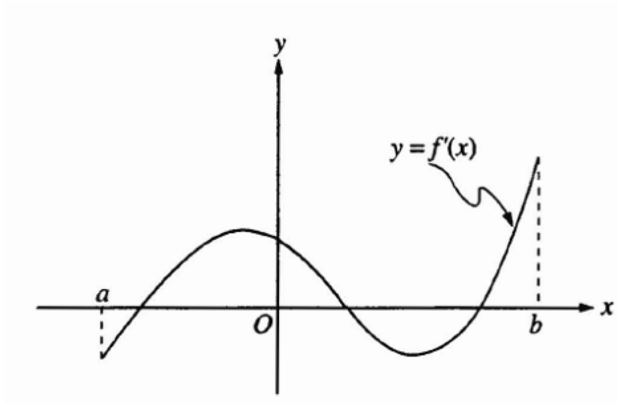
b. II only

c. III only

d. I and II only

e. II and III only

5. **1997 #12 (BC) - No Calc:**



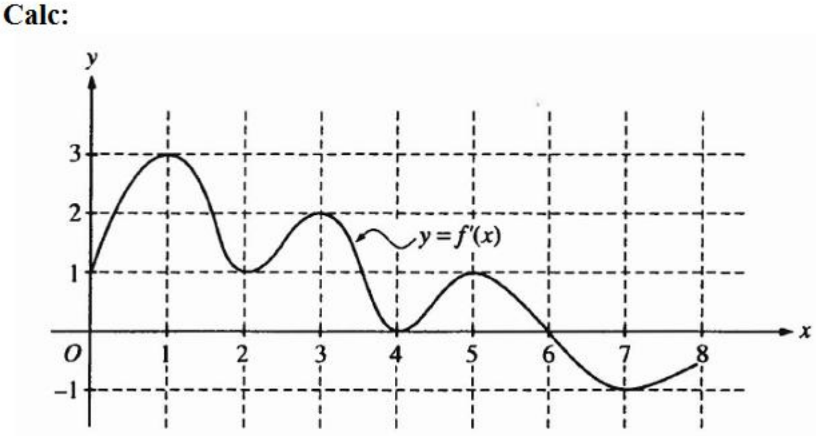
The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

a. One relative maximum and two relative minima
b. Two relative maxima and one relative minimum
c. Three relative maxima and one relative minimum
d. One relative maximum and three relative minima
e. Three relative maxima and two relative minima

6. **1997 #3 (BC) - No Calc:** The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

a. -1
b. $\frac{-\sqrt{5}}{5}$
c. 0
d. $\frac{\sqrt{5}}{5}$
e. 1

7. **1997 #9 (BC) - No Calc:**



The function f is defined on the closed interval $[0, 8]$. The graph of its derivative f' is shown above. At what value of x does the absolute minimum of f occur?

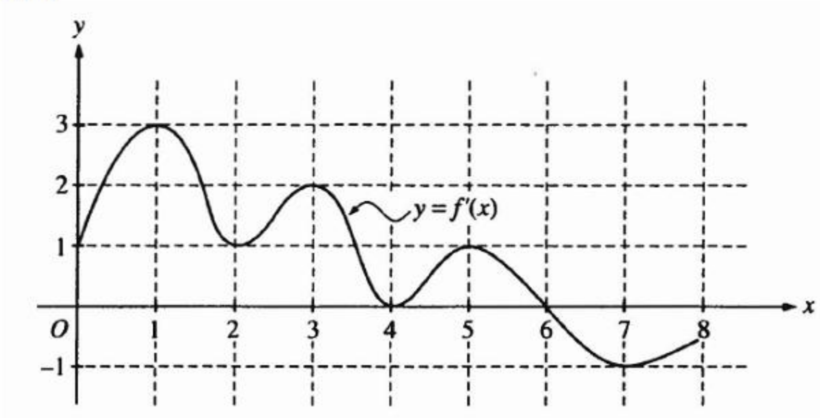
a. 0
b. 2
c. 4
d. 6
e. 8

8. **1998 #16 (BC) - No Calc:** If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

a. -1 c. 1 e. -1, 0, and 1

b. 0 d. 0 and 1

9. **1997 #8 (BC) - No Calc:**



The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above. How many points of inflection does the graph of f have?

a. Two b. Three c. Four d. Five e. Six

10. **2003 #86 (BC) - Calc OK:** Let f be the function with derivative defined by $f'(x) = \sin(x^3)$ on the interval $-1.8 < x < 1.8$. How many points of inflection does the graph of f have on this interval?

a. Two b. Three c. Four d. Five e. Six

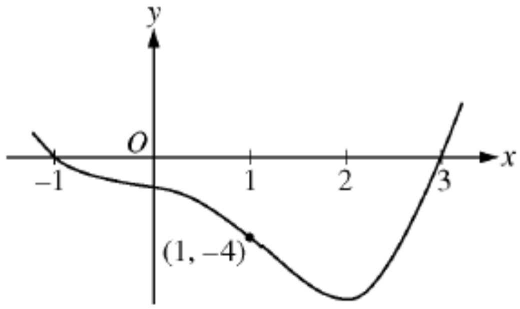
11. **1997 #80 (BC) - Calc OK:** Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x at which the graph of f changes concavity?

a. 0.56 b. 0.93 c. 1.18 d. 2.38 e. 2.44

Solutions:

1. E
2. C
3. D
4. B
5. A
6. A
7. A
8. C
9. E
10. C
11. B

❸ 2009B #5 (AB & BC) a,b,c – No Calc



Graph of f'

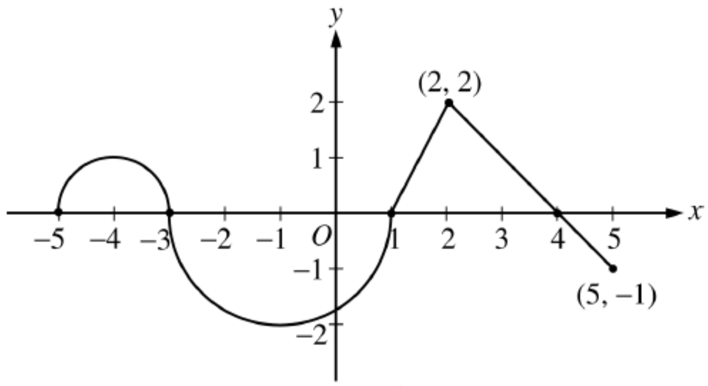
Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[\left(f'(x)\right)^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

❹ 2007B #4 (AB & BC) – No Calc



Graph of f'

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

❺ 2008 #5 (BC) a,b – No Calc

The derivative of a function f is given by $f'(x) = (x - 3)e^x$ for $x > 0$, and $f(1) = 7$.

(a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

(b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

❼ 2004 #4 (AB & BC) a,c – No Calc

Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.

❸ 2009B #5 (AB & BC) a,b,c – No Calc – Scoring Guidelines:

(a) $g(1) = e^{f(1)} = e^2$ $g'(x) = e^{f(x)} f'(x),\ g'(1) = e^{f(1)} f'(1) = -4e^2$ The tangent line is given by $y = e^2 - 4e^2(x-1)$.	$3: \begin{cases} 1: g'(x) \\ 1: g(1) \text{ and } g'(1) \\ 1: \text{tangent line equation} \end{cases}$
(b) $g'(x) = e^{f(x)} f'(x)$ $e^{f(x)} > 0$ for all x So, g' changes from positive to negative only when f' changes from positive to negative. This occurs at $x = -1$ only. Thus, g has a local maximum at $x = -1$.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{justification} \end{cases}$
(c) $g''(-1) = e^{f(-1)}\left[(f'(-1))^2 + f''(-1)\right]$ $e^{f(-1)} > 0$ and $f'(-1) = 0$ Since f' is decreasing on a neighborhood of -1, $f''(-1) < 0$. Therefore, $g''(-1) < 0$.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{justification} \end{cases}$

❹ 2007B #4 (AB & BC) – No Calc – Scoring Guidelines:

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$
f' changes from positive to negative at -3 and 4.
Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1,$ and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1,$ and 2.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{explanation} \end{cases}$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

$3 : \begin{cases} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{cases}$

❺ 2008 #5 (BC) a,b – No Calc – Scoring Guidelines:

(a) $f'(x) < 0$ for $0 < x < 3$ and $f'(x) > 0$ for $x > 3$

Therefore, f has a relative minimum at $x = 3$.

$2 : \begin{cases} 1\text{: minimum at } x = 3 \\ 1\text{: justification} \end{cases}$

(b) $f''(x) = e^x + (x-3)e^x = (x-2)e^x$
$f''(x) > 0$ for $x > 2$

$f'(x) < 0$ for $0 < x < 3$

Therefore, the graph of f is both decreasing and concave up on the interval $2 < x < 3$.

$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer with reason} \end{cases}$

❼ 2004 #4 (AB & BC) a,c – No Calc – Scoring Guidelines:

(a) $2x + 8yy' = 3y + 3xy'$

$(8y - 3x)y' = 3y - 2x$

$y' = \dfrac{3y - 2x}{8y - 3x}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{solves for } y' \end{cases}$

(c) $\dfrac{d^2y}{dx^2} = \dfrac{(8y - 3x)(3y' - 2) - (3y - 2x)(8y' - 3)}{(8y - 3x)^2}$

At $P = (3, 2)$, $\dfrac{d^2y}{dx^2} = \dfrac{(16 - 9)(-2)}{(16 - 9)^2} = -\dfrac{2}{7}$.

Since $y' = 0$ and $y'' < 0$ at P, the curve has a local maximum at P.

$4 : \begin{cases} 2 : \dfrac{d^2y}{dx^2} \\ 1 : \text{value of } \dfrac{d^2y}{dx^2} \text{ at } (3, 2) \\ 1 : \text{conclusion with justification} \end{cases}$