

Practice  
AP Calc AB

2. A particle moves along the  $x$ -axis with velocity given by  $v(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3}$  for time  $0 \leq t \leq 3.5$ .

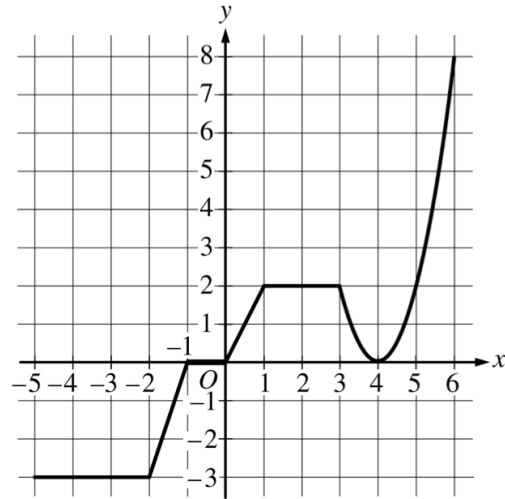
The particle is at position  $x = -5$  at time  $t = 0$ .

(a) Find the acceleration of the particle at time  $t = 3$ .

(b) Find the position of the particle at time  $t = 3$ .

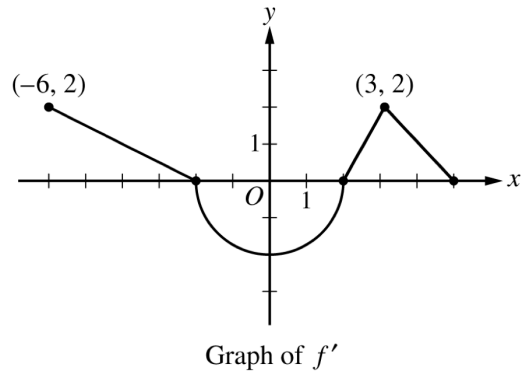
(c) Evaluate  $\int_0^{3.5} v(t) dt$ , and evaluate  $\int_0^{3.5} |v(t)| dt$ . Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the  $x$ -axis with position given by  $x_2(t) = t^2 - t$  for  $0 \leq t \leq 3.5$ . At what time  $t$  are the two particles moving with the same velocity?



Graph of  $g$

3. The graph of the continuous function  $g$ , the derivative of the function  $f$ , is shown above. The function  $g$  is piecewise linear for  $-5 \leq x < 3$ , and  $g(x) = 2(x - 4)^2$  for  $3 \leq x \leq 6$ .
- If  $f(1) = 3$ , what is the value of  $f(-5)$  ?
  - Evaluate  $\int_1^6 g(x) dx$ .
  - For  $-5 < x < 6$ , on what open intervals, if any, is the graph of  $f$  both increasing and concave up? Give a reason for your answer.
  - Find the  $x$ -coordinate of each point of inflection of the graph of  $f$ . Give a reason for your answer.



3. The function  $f$  is differentiable on the closed interval  $[-6, 5]$  and satisfies  $f(-2) = 7$ . The graph of  $f'$ , the derivative of  $f$ , consists of a semicircle and three line segments, as shown in the figure above.
- Find the values of  $f(-6)$  and  $f(5)$ .
  - On what intervals is  $f$  increasing? Justify your answer.
  - Find the absolute minimum value of  $f$  on the closed interval  $[-6, 5]$ . Justify your answer.
  - For each of  $f''(-5)$  and  $f''(3)$ , find the value or explain why it does not exist.

4. At time  $t = 0$ , a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius ( $^{\circ}\text{C}$ ) at time  $t = 0$ , and the internal temperature of the potato is greater than  $27^{\circ}\text{C}$  for all times  $t > 0$ . The internal temperature of the potato at time  $t$  minutes can be modeled by the function  $H$  that satisfies the differential equation  $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$ , where  $H(t)$  is measured in degrees Celsius and  $H(0) = 91$ .
- (a) Write an equation for the line tangent to the graph of  $H$  at  $t = 0$ . Use this equation to approximate the internal temperature of the potato at time  $t = 3$ .
- (b) Use  $\frac{d^2H}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time  $t = 3$ .
- (c) For  $t < 10$ , an alternate model for the internal temperature of the potato at time  $t$  minutes is the function  $G$  that satisfies the differential equation  $\frac{dG}{dt} = -(G - 27)^{2/3}$ , where  $G(t)$  is measured in degrees Celsius and  $G(0) = 91$ . Find an expression for  $G(t)$ . Based on this model, what is the internal temperature of the potato at time  $t = 3$  ?
-