HW solution March 18, 2020

① Find the intersection

$f(x) = g(x)$

$x = 1.033$

② Area of R

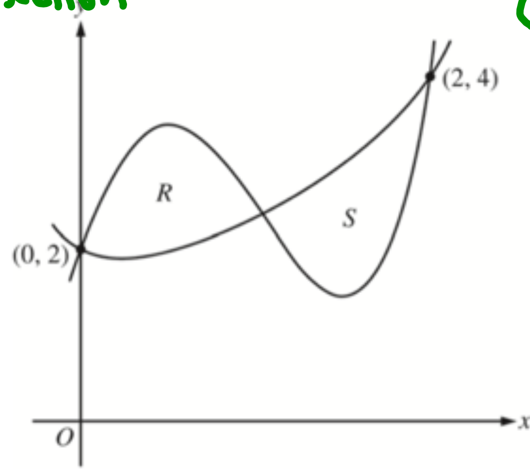
$= \int_{0}^{1.033} g(x) - f(x)\,dx$

$= .9974$

③ S

$= \int_{1.033}^{2} f(x) - g(x)\,dx$

$= 1.0069$



2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

(a) Find the sum of the areas of regions R and S.

(Calculator active)

$R + S = 2.004$

a) rel max.

f': $+ \to -$

$x = -2$

b) concave down

f' is dec.

f decreasing

$f' < 0$

$-2 < x < -1, \quad 1 < x < 3$

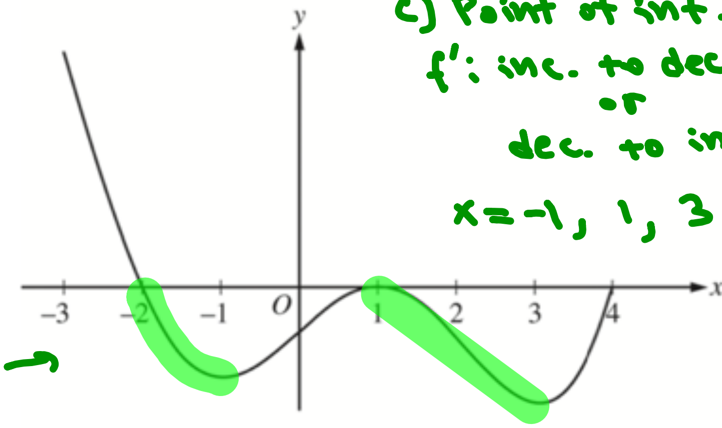
c) Point of inf.

f': inc. to dec.

or

dec. to inc.

$x = -1, 1, 3$



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

   (a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

   (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

   (c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

   (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

d) $$\int_1^x f'(t)\,dt = f(x) - f(1)$$

$$f(x) = \int_1^x f'(t)\,dt + f(1) = \int_1^x f'(t)\,dt + 3$$

$$f(4) = \int_1^4 f'(t)\,dt + 3 = 12 + 3 = 15$$

$$f(-2) = \int_1^{-2} f'(t)\,dt + 3 = -\int_{-2}^{1} f'(t)\,dt + 3$$

$$= -9 + 3 = -6$$