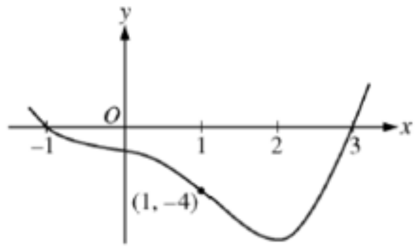
❸ 2009B #5 (AB & BC) a,b,c – No Calc

$f'' \rightarrow$ slope of f'



Graph of f'

$$g' = \underbrace{e^{f(x)}}_{\oplus}\left(\underline{f'(x)}\right)$$

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

$g' = 0 \Rightarrow x = -1$ or 3

$g' : + \rightarrow -$

g': $+$ | -1 | $-$ | 3 | $+$

Local max at $x=-1$ b/c g' about $x=-1$ changes from $+$ to $-$ as x increases.

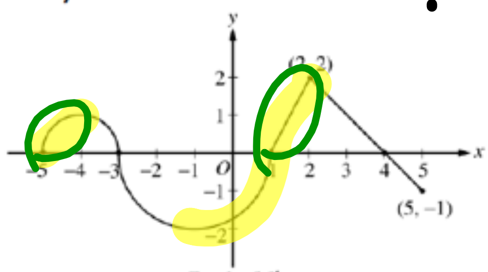
$$g''(-1) = \underbrace{e^{f(-1)}}_{\oplus}\left[\underbrace{(f'(-1))^2}_{0} + \underbrace{f''(-1)}_{\ominus}\right]$$

$f'(-1) = 0$

$f''(-1) < 0$ b/c slope of $f'(x)$ at $x=-1$ is negative

$e^{f(x)} > 0$. therefore $g''(-1) < 0$.

❹ 2007B #4 (AB & BC) – No Calc



Graph of f'

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

a) $x = -3, 4$ b/c f' changes from + to − as x increases.

b) $x = -4, -1, 2$ b/c signs of slopes of f' change, which implies f'' changes sighs.

c) f concave up → f' increases

f positive slope → f' positive. ↳ f' is above x-axis.

$-5 < x < -4$ or $1 < x < 2$

❺ 2008 #5 (BC) a,b – No Calc

$f'=0$

The derivative of a function f is given by $f'(x) = (x-3)e^x$ for $x > 0$, and $f(1) = 7$.

(a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

(b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

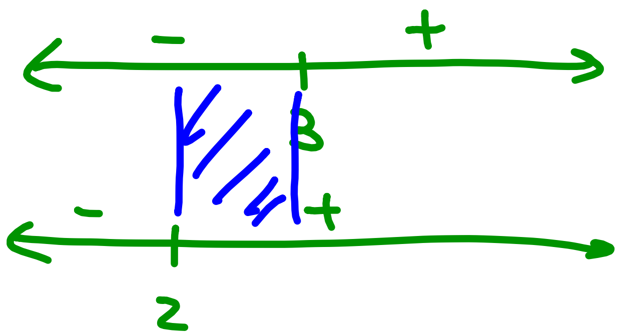
$$f'' = 1\cdot e^x + (x-3)e^x$$
$$= e^x(1+x-3) = e^x(x-2)\Big|_{x=3} = e^3\cdot 1 > 0$$

a) since $f'(3)=0$ & $f''(3)>0$,

there is rel. min at $x=3$.

b) $f'=(x-3)e^x$

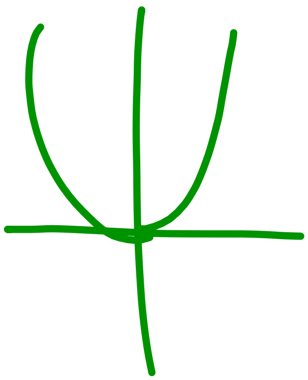
$f''=(x-2)e^x$



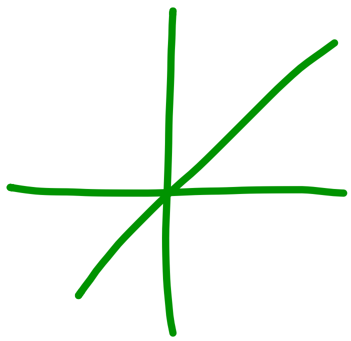
$2<x<3$ b/c f' is negative & f'' is positive in the int.

December 3, 2019

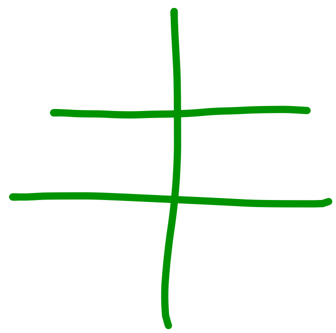
$y = x^2$



$y' = 2x$



$y'' = 2$



December 3, 2019