7. $\sqrt[3]{1-x} \approx 1 - \frac{1}{3}x$

$=$

$\downarrow$

$y \to x=0$

$(0, 1)$

$y = \sqrt[3]{1-x} = (1-x)^{1/3}$

$y' = \frac{1}{3}(1-x)^{-2/3}(-1)\Big|_{x=0} = -\frac{1}{3}$

eq. of tangent line

$y - 1 = -\frac{1}{3}(x-0)$

$y = 1 - \frac{1}{3}x$

25. $(8.06)^{2/3}$

$y = x^{2/3}$, $x=8$, $y=4$

$y' = \frac{2}{3}x^{-\frac{1}{3}}$

$= \frac{2}{3\sqrt[3]{x}}\Big|_{x=8} = \frac{1}{3}$

$y - 4 = \frac{1}{3}(x-8)$

$y = 4 + \frac{1}{3}(x-8)\Big|_{x=8.06}$

$= 4 + \frac{1}{3}(.06) = 4.02$

35. The circumference of a sphere was measured to be 84 cm with a possible error of 0.5 cm.

(a) Use differentials to estimate the maximum error in the calculated surface area. What is the relative error?

(b) Use differentials to estimate the maximum error in the calculated volume. What is the relative error?

$r = \frac{84}{2\pi} = \frac{42}{\pi}$ $\quad 2\pi r = 84$ $\quad \Delta C = .5$

$C = 2\pi r$

$\frac{dC}{dr} = 2\pi$

$dC = 2\pi\, dr$

$.5 = 2\pi\, dr$

$\frac{.5}{2\pi} = dr$

$SA = 4\pi r^2$

$\frac{dSA}{dr} = 8\pi r$

$dSA = 8\pi r\, dr$

$= \frac{4(84)(.5)}{2\pi}$

$dSA = \frac{84}{\pi}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$dV = 4\pi r^2 dr = \cancel{4}\pi\left(\frac{42}{\pi}\right)^2 \frac{.5}{\cancel{2\pi}} = \frac{1764}{\pi^2}$

33. The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error, relative error, and percentage error in computing (a) the volume of the cube and (b) the surface area of the cube.

$e = 30$

$\Delta e = .1$

$V = e^3$

$\frac{dV}{de} = 3e^2$

$dV = 3e^2 de = 3(30)^2(.1)$

$= 270\ cm^3$

$SA = 6e^2$

$dSA = 12e\, de$

$= 12(30)(.1) = 36\ cm^2$

17. $y = \tan x, \quad x = \pi/4, \quad dx = -0.1$

$dy = \sec^2 x \, dx \Big|_{\substack{x=\pi/4 \\ dx=-.1}} = \sec^2\left(\frac{\pi}{4}\right)(-.1)$

$= 2(-.1)$

$= -.2$

23. $(2.001)^5$

$y = x^5, \quad x=2 \rightarrow y=32$

$y' = 5x^4 = 5(16) = 80$

$y - 32 = 80(x-2)$

$y = 32 + 80(x-2) \Big|_{x=2.001} = 32 + 80(.001)$

$= 32.08$