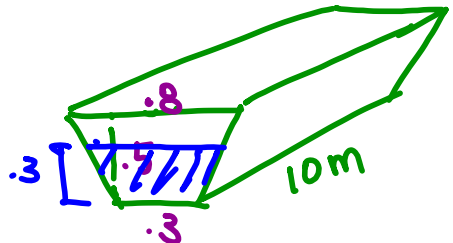
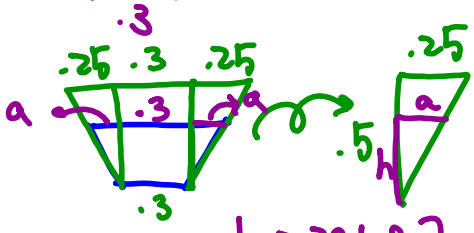
25. A water trough is 10 m long and a cross-section has the shape of an isosceles trapezoid that is 30 cm wide at the bottom, 80 cm wide at the top, and has height 50 cm. If the trough is being filled with water at the rate of 0.2 m^3/min, how fast is the water level rising when the water is 30 cm deep?



$V = \frac{(b_1+b_2)h}{2}(H)$

$= \frac{(.3+b_2)h}{2}(10)$



$\frac{h}{a} = \frac{.5}{.25} = 2$

$b_2 = 2a + .3$

$= h + .3$

$h = 2a$

$V = \frac{(.3+.3+h)h}{2}(10)$

$= 5h(h+.6)$

$V = 5h^2 + 3h$

$\frac{dV}{dt} = 10h\frac{dh}{dt} + 3\frac{dh}{dt}$

$.2 = 10(.3)\frac{dh}{dt} + 3\frac{dh}{dt}$

$.2 = 6\frac{dh}{dt}$

$\frac{1}{30} = \frac{dh}{dt}$

water level is increasing at the rate of $\frac{1}{30}$ m/min.

34. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

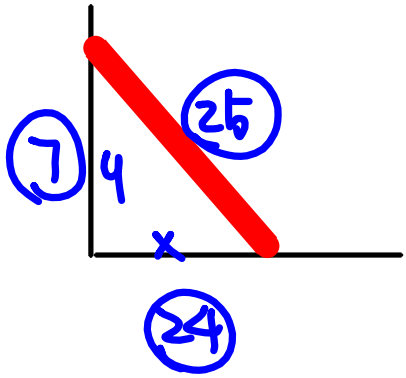
(A) $-\frac{7}{8}$ feet per minute

(B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute

(D) $\frac{7}{8}$ feet per minute

(E) $\frac{21}{25}$ feet per minute



$\frac{dy}{dt} = -3 \text{ ft/min}$

$\frac{dx}{dt} = ?$

$x^2 + y^2 = 25^2$

$2xx' + 2yy' = 0$

$xx' + yy' = 0$

$24x' + 7(-3) = 0$

$x' = \frac{21}{24} = \frac{7}{8}$

9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

$\frac{dA}{dt} = 2\frac{dr}{dt}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$2\frac{dr}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{1}{\pi} = r$ ← $1 = \pi r$

10. If $y = 10^{(x^2-1)}$, then $\frac{dy}{dx} =$ $10^{x^2-1} \cdot \ln 10 \cdot (2x)$

(A) $(\ln 10)10^{(x^2-1)}$

(B) $(2x)10^{(x^2-1)}$

(C) $(x^2-1)10^{(x^2-2)}$

(D) $2x(\ln 10)10^{(x^2-1)}$

(E) $x^2(\ln 10)10^{(x^2-1)}$

12. The position of a particle moving along the x-axis is $x(t) = \sin(2t) - \cos(3t)$ for time $t \geq 0$. When $t = \pi$, the acceleration of the particle is

(A) 9 (B) $\frac{1}{9}$ (C) 0 (D) $-\frac{1}{9}$ (E) -9

$x' = 2\cos(2t) + 3\sin(3t)$

$a = x'' = -4\sin(2t) + 9\cos(3t)\Big|_{t=\pi}$

$= 0 + 9(-1) = -9$

16. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

$V=0$

$x' = V = 0 = 3t^2 - 6t - 9$

$0 = t^2 - 2t - 3$

$(t-3)(t+1)$

$t = 3, \cancel{-1}$

22. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

(A) 6 (B) 8 (C) 16 (D) $4\sqrt{3}$ (E) $12\sqrt{3}$

$\frac{dA}{dt} = 96\pi \text{ m}^2/\text{sec}$

$A = 64\pi \rightarrow r = 8$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$96\pi = 2(\pi)(8)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{96\pi}{16\pi}$

$= 6 \text{ m/sec}$

25. A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$. For what values of t is the particle at rest?

(A) No values (B) 0 only (C) $\frac{1}{2}$ only (D) 1 only (E) 0 and $\frac{1}{2}$

$$x' = v = 0 = e^{-2t} + t(-2e^{-2t})$$

$$= e^{-2t}(1 - 2t)$$

$$t = \frac{1}{2}$$

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25. $\frac{d}{dx}\left(2^x\right) =$ $2^x \ln 2$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

$\frac{dr}{dt} = .3$

$\frac{dV}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 100\pi(.3) = 30\pi$