19. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?

$\frac{dA}{dt} = 2\ \text{cm}^2/\text{min}$ $\quad \frac{db}{dt} = ?$ $\quad \frac{dh}{dt} = 1\ \text{cm}/\text{min}$

$A = \frac{1}{2}bh \rightarrow 100 = \frac{1}{2}b(10) \rightarrow b = 20$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt}h + \frac{dh}{dt}b\right)$

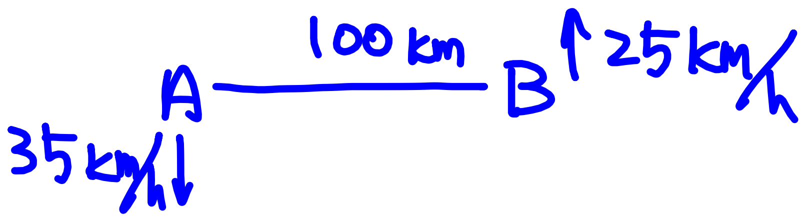
$2 = \frac{1}{2}\left(\frac{db}{dt}(10) + 1(20)\right)$

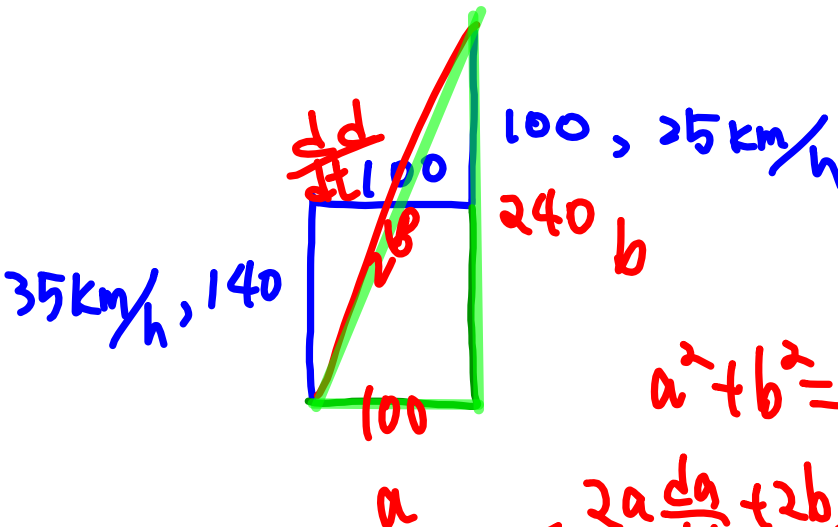
$-16 = 10\frac{db}{dt}$

$-1.6\ \text{cm}/\text{min} = \frac{db}{dt}$

base is decreasing at the rate of 1.6 cm/min.

21. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?





$$a^2+b^2=d^2$$

$$2a\frac{da}{dt}+2b\frac{db}{dt}=2\frac{dd}{dt}d$$

$$aa'+bb'=dd'$$

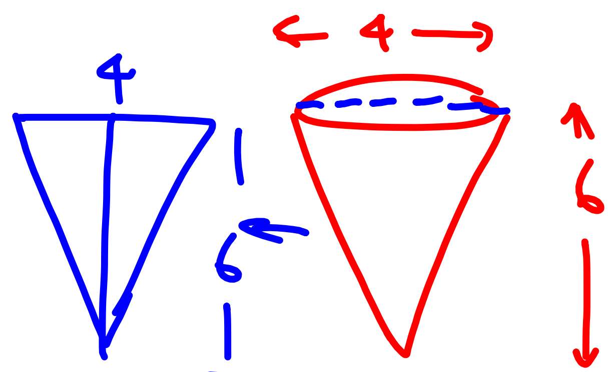
$$100\cdot(0)+240(60)=260d'$$

$$\frac{720}{13}\text{ km/h}=\frac{\cancel{240}^{12}\cdot 60}{\cancel{260}_{13}}=d'$$

23. Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

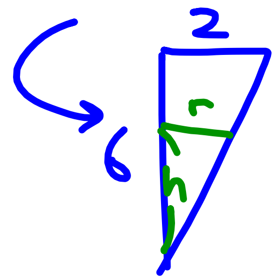
$$\frac{dV}{dt} = -10000\ cm^3/min \qquad \frac{dh}{dt} = 20\ cm/min \qquad \frac{dV}{dt} = ?$$

$$V = \frac{1}{3}\pi r^2 h$$



$$V = \frac{1}{3}\pi r^2 (3r)$$

$$V = \pi r^3$$



$$\frac{r}{2} = \frac{h}{6}$$

$$3r = h$$

$$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$$

$$3r = h \qquad 3\frac{dr}{dt} = \frac{dh}{dt}$$

$$3r = 2$$

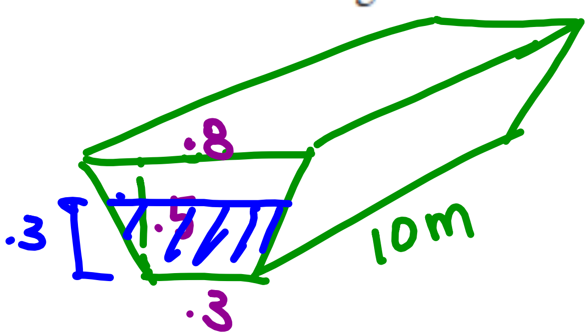
$$r = \frac{2}{3} \qquad \frac{dr}{dt} = \frac{20}{3}$$

$$\frac{dV}{dt} = 3\pi\left(\frac{2}{3}\right)^2 \frac{20}{3}$$

$$= 27.925$$

$$\frac{dV_{pump}}{dt} = 10027.925\ cm^3/min$$

25. A water trough is 10 m long and a cross-section has the shape of an isosceles trapezoid that is 30 cm wide at the bottom, 80 cm wide at the top, and has height 50 cm. If the trough is being filled with water at the rate of 0.2 m^3/min, how fast is the water level rising when the water is 30 cm deep?



$$V=\frac{(b_1+b_2)h}{2}(H)$$

$$=\frac{(.3+b_2)h}{2}(10)$$